Topic Y2: Coordinate geometry and binomial (Post-TT B) [40] MARKSCHEME

1.

	$(3x-2)^4 = 81x^4 - 216x^3 + 216x^2 - 96x + 16$	M1 A1 A1 A1	 4 **4**	Attempt binomial expansion, including attempt at coeffs. Obtain one correct, simplified, term Obtain a further two, simplified, terms Obtain a completely correct expansion

2.

(i)		**B1**	$(x-2)^2$ and $(y-1)^2$ seen
	$(x-2)^2 + (y-1)^2 = 100$	**B1**	$(x \pm 2)^2 + (y \pm 1)^2 = 100$
	$x^2 + y^2 - 4x - 2y - 95 = 0$	**B1**	correct form
		3	
(ii)	$(5-2)^2 + (k-1)^2 = 100$	**M1**	$x = 5$ substituted into their equation
	$(k-1)^2 = 91$ or $k^2 - 2k - 90 = 0$	**A1**	correct, simplified quadratic in k (or y) obtained
	$k = 1 + \sqrt{91}$	**A1**	cao
		3	
(iii)	distance from (–3, 9) to (2, 1)		
	$= \sqrt{(2--3)^2 + (1-9)^2}$	**M1**	Uses $(x_2 - x_1)^2 + (y_2 - y_1)^2$
	$= \sqrt{25+64}$	**A1**	
	$= \sqrt{89}$		
	$\sqrt{89} < 10$ so point is inside	**B1**	compares their distance with 10 and makes consistent conclusion
		3	
(iv)	gradient of radius $= \frac{9-1}{8-2}$	**M1**	uses $\frac{y_2 - y_1}{x_2 - x_1}$
	$= \frac{4}{3}$	**A1**	oe
	gradient of tangent $= -\frac{3}{4}$	**B1√**	oe
	$y - 9 = -\frac{3}{4}(x-8)$	**M1**	correct equation of straight line through (8, 9), any non-zero gradient
	$y - 9 = -\frac{3}{4}x + 6$		
	$y = -\frac{3}{4}x + 15$	**A1**	oe 3 term equation
		5	

3.

3 (i)	$(1 + \frac{1}{2}x)^{10} = 1 + 5x + 11.25x^2 + 15x^3$	**B1**		Obtain $1 + 5x$
		M1		Attempt at least the third (or fourth) term of the binomial expansion, including coeffs
		A1		Obtain $11.25x^2$
		A1		Obtain $15x^3$
			4	
(ii)	coeff of $x^3 = (3 \times 15) + (4 \times 11.25) + (2 \times 5)$ $= 100$	**M1**		Attempt at least one relevant term, with or without powers of x
		A1 ft		Obtain correct (unsimplified) terms (not necessarily summed) – either coefficients or still with powers of x involved
		A1	**3**	Obtain 100
			7	

4.

(i)	Centre (4, 1)	B1		Correct centre	
	$(x-4)^2 + (y-1)^2 - 16 - 1 - 3 = 0$	M1		Correct method to find r^2	$r^2 = (\pm \text{ their } 4)^2 + (\pm \text{their } 1)^2 + 3$ soi
	$(x-4)^2 + (y-1)^2 = 20$				
	Radius $= \sqrt{20}$	A1	3	Correct radius	$\pm\sqrt{20}$ is **A0** Ignore incorrect simplification of $\sqrt{20}$
(ii)	$k = 1 \pm \sqrt{20}$	M1 A1ft		y ordinate of their centre ± their radius or Both correct, unsimplified values	Alternatives for method mark : a) Substitutes k for y and uses $b^2 - 4ac = 0$ to obtain quadratic in k b) Recognises $x = 4$ is equation of normal, substitutes into circle equation and solves for k. **SR** $k = 1 + \sqrt{20}$ or $k = 1 - \sqrt{20}$ or better www **B1**
	$k = 1 \pm 2\sqrt{5}$	A1	3	**cao**	
(iii)	$MT^2 = r^2 - 2^2$	M1		Correct use of Pythagoras' theorem involving MT (or SM)	**SR** ST=8 from particular S and T co-ordinates [e.g. horizontal chord calculated as (0,3) and (8,3)] **B1** Justifies solution the same for all possible chords **B2**
	$MT = 4$	A1ft		Correct value of MT for their r	
	$ST = 8$	A1	3	**cao**	
(iv)	$x = 2y + 12$	M1*		Attempt to solve equations simultaneously	Must be a clear attempt to reduce to one variable using equation of line and either form of equation of circle. Condone poor algebra for first mark.
	$(2y+8)^2 + (y-1)^2 = 20$	A1		Correct unsimplified expression, may be $(12+2y)^2 + y^2 - 8(12+2y) - 2y - 3 = 0$	If y eliminated:
	$4y^2 + 32y + 64 + y^2 - 2y + 1 = 20$				
	$5y^2 + 30y + 45 = 0$	A1		Obtain correct 3 term quadratic	$(x-4)^2 + \left(\frac{1}{2}x - 7\right)^2 = 20$
	$y^2 + 6y + 9 = 0$				
	$(y+3)^2 = 0$	DM1		Correct method to solve quadratic of form $ax^2 + bx + c = 0$ $(b \neq 0)$	Or $x^2 + \left(\frac{1}{2}x - 6\right)^2 - 8x - 2\left(\frac{1}{2}x - 6\right) - 3 = 0$
	$y = -3$	A1		y value correct, no extra solutions	
	$x = 6$	A1		x value correct **ISW**	Leading to $x^2 - 12x + 36 = 0$
	OR				
	$y - 1 = -2(x - 4)$	M1		Attempt to find equation of radius/normal	
		A1		Correct equation	
	Solve simultaneously with $y = \frac{1}{2}x - 6$	M1			
	$x = 6$	A1			
	$y = -3$	A1	6		**SR** Correct coordinates spotted or from trial and improvement www **B2**
	States line is tangent as meets at one point or verifies (6, -3) lies on circle	B1	15	Allow showing distance between (6,-3) and (4,1) = $\sqrt{20}$	