Topic Y3 Trigonometry (Pre-TT) [45] MARKSCHEME

1.

(i) area = $\frac{1}{2}\times 5\sqrt{2}\times 8\times \sin 60^\circ$	B1		State or imply that $\sin 60^\circ = \frac{\sqrt{3}}{2}$ or exact equiv
$= \frac{1}{2}\times 5\sqrt{2}\times 8\times \frac{\sqrt{3}}{2}$	M1		Use $\frac{1}{2}ac\sin B$
$= 10\sqrt{6}$	A1	3	Obtain $10\sqrt{6}$ only, from working in surds
(ii) $AC^2 = (5\sqrt{2})^2 + 8^2 - 2\times 5\sqrt{2}\times 8\times \cos 60^\circ$	M1		Attempt to use the correct cosine formula
	A1		Correct unsimplified expression for AC^2
$AC = 7.58$ cm	A1	3	Obtain $AC = 7.58$, or better
		6	

2.

(i)	$\sin^2 x = 1-\cos^2 x \Rightarrow 2\cos^2 x + \cos x - 1 = 0$	M1		For transforming to a quadratic in cos *x*
	Hence $(2\cos x - 1)(\cos x + 1) = 0$	M1		For solution of a quadratic in cos *x*
	$\cos x = \frac{1}{2} \Rightarrow x = 60^\circ$	A1		For correct answer 60°
	$\cos x = -1 \Rightarrow x = 180^\circ$	A1	4	For correct answer 180° [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2
(ii)	$\tan 2x = -1 \Rightarrow 2x = 135$ or 315	M1 M1		For transforming to an equation of form tan2*x* = *k* For correct solution method, i.e. inverse tan followed by division by 2
	Hence $x = 67.5^\circ$ or 157.5°	A1 A1	4	For correct value 67.5 For correct value 157.5
	OR $\sin^2 2x = \cos^2 2x$ $2\sin^2 2x = 1$ $\quad 2\cos^2 2x = 1$ $\sin 2x = \pm\frac{1}{2}\sqrt{2}$ $\quad \cos 2x = \pm\frac{1}{2}\sqrt{2}$ Hence $x = 67.5^\circ$ or 157.5°	M1 M1 A1 A1		Obtain linear equation in cos 2*x* or sin 2*x* Use correct solution method For correct value 67.5 For correct value 157.5 [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2
			8	

3.

Question	Scheme	Marks	AOs
6(i)	Tries at least one value in the interval Eg $4^2 - 4 - 1 = 11$	M1	1.1b
	States that when $n = 8$ it is FALSE and provides evidence $8^2 - 8 - 1 = 55 = (11\times 5)$ Hence NOT PRIME	A1	2.4
		(2)	
(ii)	Knows that an odd number is of the form $2n+1$	B1	3.1a
	Attempts to simplify $(2n+1)^3 - (2n+1)^2$	M1	2.1
	……….and factorise $8n^3 + 8n^2 + 2n = 2(4n^3 + 4n^2 + 1n) =$	dM1	1.1b
	with statement $2\times ..$ is always even	A1	2.4
		(4)	

4.

(i)		$b^2 = 2.4^2 + 2^2 - 2 \times 2.4 \times 2 \times \cos 40°$ $b = 1.55$ km	M1	Attempt use of correct cosine rule	Must be correct formula seen or implied, but allow slip when evaluating eg omission of 2, incorrect extra 'big bracket' Allow M1 even if subsequently evaluated in rad mode (4.02) Allow M1 if expression is not square rooted, as long as LHS was intended to be correct ie $b^2 = \ldots$ or $AC^2 = \ldots$
			A1 [2]	Obtain 1.55, or better	Actual answer is 1.55112003… so allow more accurate answer as long as it rounds to 1.551 Units not required
(ii)		$\frac{\sin A}{2} = \frac{\sin 40}{1.55}$ $\frac{\sin C}{2.4} = \frac{\sin 40}{1.55}$ $A = 56°$ $C = 84°$	M1	Attempt to find one of the other two angles in triangle	Could use sine rule or cosine rule, but must be correct rule attempted Need to substitute in and rearrange as far as $\sin A = \ldots$ / $\cos A = \ldots$ etc, but may not actually attempt angle
		hence bearing is 124°	A1	Obtain $A = 56°$, or $C = 84°$	Any angle rounding to 56° or 84°, and no errors seen
			A1ft [3]	Obtain 124°, following their angle A or C	Allow any answer rounding to 124 Finding bearing of A from C is A0 – ie not a MR
(iii)		$d = 2 \times \sin 40°$ $= 1.29$ km	M1	Attempt perpendicular distance	Any valid method, but must attempt required distance Can still get M1 if using incorrect or inaccurate sides / angles found earlier in question Allow M1 if evaluated in rad mode (1.49)
			A1 [2]	Obtain 1.29, or better	Allow more accurate final answers in range [1.285, 1.286] A0 for inaccurate answers due to PA elsewhere in question (typically $C = 84.4$, so $A = 55.6$, so $d = 1.28$) Units not required

5.

(i)		**DR** $BE = \sqrt{3}$ from the standard triangle BDE	B1	2.2a	Or $AB = 1 + \sqrt{3}$ seen	**B0** for decimal
		$BC = AB\cos 45$	M1	2.1	oe or Pythagoras' theorem	Must be seen
		$BC = \frac{1+\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2}$	E1 [3]	2.2a	AG	$\frac{1+\sqrt{3}}{\sqrt{2}}$ must be seen
(ii)		**DR** Triangle ABC is isosceles so $BC = AC$ but $AC = CD + \sqrt{2}$	B1	2.4	State or imply that $BC = AC$ and state $AC = CD + \sqrt{2}$	
		so $CD = \frac{\sqrt{2}+\sqrt{6}}{2} - \sqrt{2}$ $= \frac{\sqrt{6}-\sqrt{2}}{2}$	M1	2.1	Obtain expression for CD, may be unsimplified	**M0** if decimals seen
		$\sin 15 = \frac{CD}{BD} = \frac{\sqrt{6}-\sqrt{2}}{2} \div 2 = \frac{\sqrt{6}-\sqrt{2}}{4}$	A1 [3]	2.2a	Obtain expression for $\sin 15$ and simplify to answer given	**SC1** for showing using addition formula

6.

(i)		$f(2) = 32 - 14 - 3 = 15$	M1	Attempt f(2) or equiv	M0 for using $x = -2$ (even if stated to be f(2)) At least one of the first two terms must be of the correct sign Must be evaluated and not just substituted Allow any other valid method as long as remainder is attempted (see guidance in part **(ii)** for acceptable methods)
			A1	Obtain 15	Do not ISW if subsequently given as -15 If using division, just seeing 15 on bottom line is fine unless subsequently contradicted by eg -15 or $^{15}/_{x-2}$
			[2]		
(ii)		$f(^{-1}/_2) = {^{-1}/_2} + {^7/_2} - 3 = 0$ **AG**	B1	Confirm $f(^{-1}/_2) = 0$, with at least one line of working	$4(^{-1}/_2)^3 - 7(^{-1}/_2) - 3 = 0$ is enough B0 for just $f(^{-1}/_2) = 0$ If, and only if, $f(^{-1}/_2)$ is **not** attempted then allow B1 for other evidence such as division / coeff matching etc If using division must show '0' on last line or make equiv comment such as 'no remainder' If using coefficient matching must show 'R = 0' Just writing $f(x)$ as the product of the three correct factors is not enough evidence on its own for B1
		$f(x) = (2x + 1)(2x^2 - x - 3)$	M1	Attempt complete division by $(2x + 1)$, or another correct factor	Could divide by $(x + 1)$, $(x + {^1/_2})$, $(2x - 3)$, $(x - {^3/_2})$ Must be complete method - ie all 3 terms attempted Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of quadratic, considering all relevant terms each time
		$= (2x + 1)(2x - 3)(x + 1)$	A1	Obtain $2x^2$ and one other correct term	Could be middle or final term depending on method Must be correctly obtained Coeff matching - allow for $A = 2$ etc Or lead term and one another correct for their factor
			A1	Obtain fully correct quotient of $2x^2 - x - 3$	Could appear as quotient in long division, or as part of a product if using inspection. For coeff matching it must now be explicit not just $A = 2$, $B = -1$, $C = -3$ Or fully correct quotient for their factor
			M1	Attempt to factorise their quadratic quotient from division attempt by correct factor	Allow M1 if brackets would give two correct terms on expansion **SR** allow even if their quadratic does not have rational roots If solving quadratic (eg using the formula) then must attempt factors for M1, but allow eg $(x - {^3/_2})(x + 1)$
			A1	Obtain $(2x + 1)(2x - 3)(x + 1)$	Final answer must be seen as a product of all three factors Allow factorised equiv such as $2(2x + 1)(x - {^3/_2})(x + 1)$ but A0 for $(2x + 1)(x - {^3/_2})(2x + 2)$ as not fully factorised isw if subsequent confusion over 'roots' and 'factors' **SR** If repeated use of factor theorem, or answer given with no working, then allow a possible B1for $f(^{-1}/_2) = 0$ with an additional **B5** for $(2x + 1)(2x - 3)(x + 1)$, or **B3** for a multiple such as $(2x + 1)(x - {^3/_2})(x + 1)$
			[6]		
(iii)		$2\cos\theta + 1 = 0$ $\cos\theta + 1 = 0$ $2\cos\theta - 3 = 0$	M1*	Identify relationship between factors of $f(\cos\theta)$ and factors of $f(x)$	Replace x with $\cos\theta$ in at least one of their factors (could be implied by later working, inc their solutions)
		$\cos\theta = {^{-1}/_2}$ $\cos\theta = -1$ $\cos\theta = {^3/_2}$	M1d*	Attempt to solve $\cos\theta = k$ at least once	Must actually attempt θ, with $-1 \le k \le 1$
		$\theta = {^{2\pi}/_3}, {^{4\pi}/_3}$ $\theta = \pi$	A1	Obtain at least 2 correct angles	Allow angles in degrees (120°, 240°, 180°) Allow decimal equivs (2.09, 4.19, 3.14) Allow if $2\cos\theta + 1 = 0$ is the only factor used, or if other incorrect factors are also used Allow M1M1A1 for 2 correct angles with no working shown
			A1	Obtain all 3 correct angles	Must be exact and in radians A0 if additional incorrect angles in range Allow full credit if no working shown Angles must come from 3 correct roots of $f(x)$, but allow if a factor was eg $(x - {^3/_2})$ not $(2x - 3)$ A0 if incorrect root, even if it doesn't affect the three solutions eg one of their factors was $(2x + 3)$ not $(2x - 3)$
			[4]		