

Area of a triangle

Starter

1. **(Review of last lesson)** A plane flies around a storm by flying 15 miles on a course of 069° and then 18 miles on a course of 295° . Calculate the distance and bearing of its original position from its new position.

Working:

$$360^\circ - 295^\circ = 65^\circ$$

$$180^\circ - 65^\circ - 69^\circ = 46^\circ$$

$$180^\circ - 65^\circ = 115^\circ$$

Cosine rule:

$$x^2 = 18^2 + 15^2 - 2 \times 18 \times 15 \times \cos 46^\circ$$

$$x \approx 13.1865$$

Sine rule:

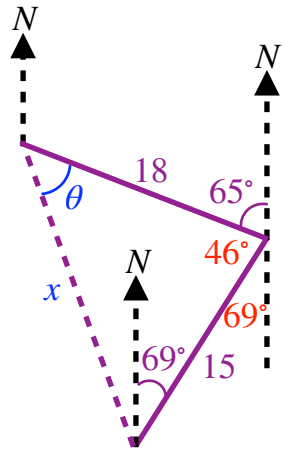
$$\frac{\sin \theta}{15} = \frac{\sin 46^\circ}{13.1865\dots}$$

$$\theta = \sin^{-1}\left(\frac{15 \sin 46^\circ}{13.1865\dots}\right)$$

$$\theta \approx 54.912^\circ$$

The required angle is $115^\circ + 54.912^\circ \approx 169^\circ$

The distance is 13.2 miles (3 s.f.) and the bearing is 169° (3 s.f.)



- E.g. 1** Two sides of a triangle are 4 cm and 5 cm. Given that the angle between the two sides is obtuse and that the area of the triangle is 8 cm^2 , calculate the size of the obtuse angle to 3 s.f..

Working:

$$\text{Area} = \frac{1}{2}ab \sin C: \quad \frac{1}{2} \times 4 \times 5 \sin C = 8$$

$$\sin C = \frac{16}{20} = \frac{4}{5}$$

$$C = 53.1^\circ$$

Since the angle is obtuse, the required angle is $180^\circ - 53.1^\circ = 127^\circ$ (3 s.f.)

E.g. 2 The points A , B and C have coordinates $(-1, -3)$, $(2, 4)$ and $(5, -7)$ respectively.

- (a) Find $\angle ABC$.
(b) Hence, or otherwise, calculate the area of triangle ABC .

Working:

(a) $AB = \sqrt{(2 - -1)^2 + (4 - -3)^2} = \sqrt{58}$
 $BC = \sqrt{(5 - 2)^2 + (-7 - 4)^2} = \sqrt{130}$
 $AC = \sqrt{(5 - -1)^2 + (-7 - -3)^2} = \sqrt{52} = 2\sqrt{13}$
 $\angle ABC$ is opposite side AC
 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$: $\cos \angle ABC = \frac{58 + 130 - 52}{2 \times \sqrt{58} \times \sqrt{130}}$
 $\angle ABC = 38.5^\circ$ (3 s.f.)

(b) Area = $\frac{1}{2}ab \sin C$:
Area of $\triangle ABC = \frac{1}{2} \times \sqrt{58} \times \sqrt{130} \times \sin 38.45^\circ = 27$

Video: [Area of a triangle using sine](#)

[Solutions to Starter and E.g.s](#)

Exercise

p216 11C Qu 1i, 2i, 3-7