Combining probabilities

Starter

1. *(Review of last lesson)* Simon plays football and tennis. He has one match in each sport to play. The probability that he will win the football match is 0.5, while the probability of losing is 0.2. He has a $65\,\%$ chance of winning the tennis match. The outcomes of the matches are independent.

(a) Draw a tree diagram showing ***all*** the possible outcomes.

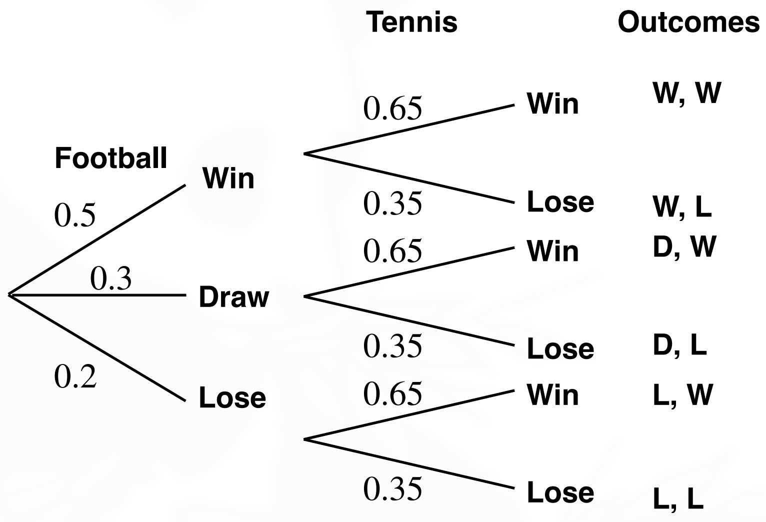
(b) Find the probability that Simon:

(i) wins both matches

(ii) wins only one match

(iii) wins the tennis match only.

Working: (a)



(b) (i) $P(\text{wins both matches}) = 0.5 \times 0.65 = 0.325$

(ii) $P(\text{wins only one match}) = P(W, L) + P(D, W) + P(L, W)$

$= 0.5 \times 0.35 + 0.3 \times 0.65 + 0.2 \times 0.65$

$= 0.5$

(iii) $P(\text{wins the tennis match only}) = P(D, W) + P(L, W)$

$= 0.3 \times 0.65 + 0.2 \times 0.65$

$= 0.325$

E.g. 1 Events X and Y are independent such that $P(X') = \frac{11}{20}$ and $P(X \cap Y) = \frac{1}{5}$. Find $P(X \cup Y)$ when:

(a) events X and Y are mutually exclusive

(b) events X and Y are not mutually exclusive.

Working: (a) $P(X) = 1 - P(X') = 1 - \frac{11}{20} = \frac{9}{20}$

$P(X \cap Y) = P(X) \times P(Y)$: $\quad \frac{1}{5} = \frac{9}{20} \times P(Y)$

$P(Y) = \frac{4}{9}$

X and Y are mutually exclusive: $\quad P(X \cup Y) = P(X) + P(Y)$

$$= \frac{9}{20} + \frac{4}{9}$$

$$= \frac{161}{180}$$

(b) X and Y are not mutually exclusive:

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$= \frac{9}{20} + \frac{4}{9} - \frac{1}{5}$$

$$= \frac{25}{36}$$

E.g. 2 Ms. Smith asked 48 students which triathlon event they watched.

9 watched all three sports
5 watched swimming and cycling only
10 watched the swimming and running
22 watched the swimming
17 watched the cycling and running
25 watched the cycling
29 watched the running

(a) Draw a Venn diagram to show this information, stating how many students watched none of the sports.

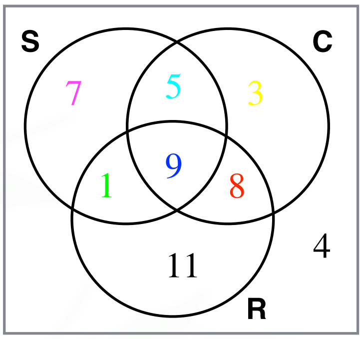
(b) A student from the class is chosen at random. Find the probability that they watched cycling and running only.

(c) A student who watched the swimming is chosen at random. Find the probability that they also watched the running but not the cycling.

(d) Two students are chosen at random. Find the probability that the first student watch the swimming and the second student watched the cycling.

Working:

(a) 9 watched all three sports — middle
5 watched S & C only
10 watched the S & R so $10 - 9 = 1$
22 watched S so $22 - 9 - 1 - 5 = 7$
17 watched the C & R so $17 - 9 = 8$
25 watched C so $25 - 9 - 8 - 5 = 3$
29 watched R so $29 - 1 - 9 - 8 = 11$
So the number who watch no sports is:
$48 - 7 - 5 - 3 - 1 - 9 - 8 - 11 = 4$



(b) $P(C \;\&\; R \text{ only}) = \dfrac{8}{48} = \dfrac{1}{6}$

(c) Only look at the swimming circle

$P(R \text{ but not } C \text{ from S}) = \dfrac{1}{22}$

(d) Consider groups of swimming watching students who didn't watch the cycling and who did watch the cycling

$P(S \text{ but not } C, C) = \dfrac{7+1}{48} \times \dfrac{5+3+9+8}{47} = \dfrac{25}{282}$

$P(S \text{ and } C, C) = \dfrac{5+9}{48} \times \dfrac{5+3+9+8-1}{47} = \dfrac{7}{47}$

Required probability $= \dfrac{25}{282} + \dfrac{7}{47} = \dfrac{67}{282}$

E.g. 3 The 150 visitors to an exhibition completed a questionnaire. One question asked whether they were a **student**, in **employment** or a **retailer**. Another question what their main method of transport was to arrive at the exhibition: **car**, **public transport** or **bike**.
From the 37 students, 9 arrived by car and 20 arrived by public transport.
Of the total of 90 visitors who arrived by car, 34 were retailers.
Only 15 attendees arrived on a bike, with 6 of these being employed.
The number of retailers who arrived on public transport was the same as the number of employees who arrived on a bike.

(a) Design a two-way table to show this information
(b) One of the visitors to the exhibition is chosen at random. Find the probability that they were a student on a bike.
(c) A retailer is selected at random. Find the probability that they arrived by car.

Working: (a)

	Car	Public transport	Bike	Total
Student	9	20	8	37
Employed	47	19	6	72
Retailer	34	6	1	41
Total	90	45	15	150

(b) $P(\text{student on a bike}) = \dfrac{8}{150} = \dfrac{4}{75}$

(c) $P(\text{by car from the retailers}) = \dfrac{34}{41}$

Video: Tree diagrams
Video: Probability trees for independent events
Video: AND and OR rule for probability (including mutually exclusive events)
Video: Probability trees for dependent events
Video: Venn diagrams
Video: Venn diagrams (3 circle problems)
Video: Venn diagrams - notation and defining regions
Video: Probability in Venn diagrams
Video: General probability formula and mutually exclusive events
Video: Two-way tables

Solutions to Starter and E.g.s

Exercise
p364 17A Qu 1i, 2-11