Connected Particles (vertical)

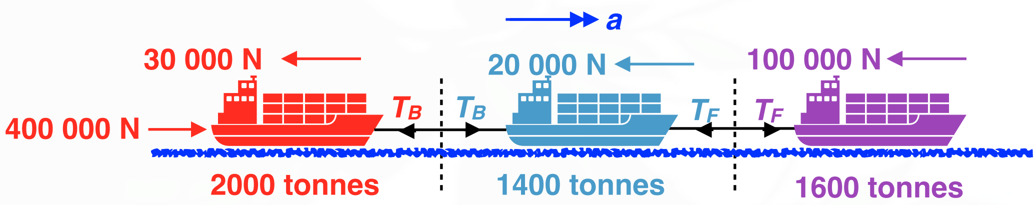
Starter

1. (Review of last lesson) Three barges travel down a river in line. Only the rear barge has an engine, which produces a forward force of $400k$ N. The masses of the barges are 1600 tonnes, 1400 tonnes and 2000 tonnes and the water exerts resistance of $100k$ N, $20k$ N and $30k$ N respectively (front to back). Find:
   (a) the acceleration of the barges

   (b) the forces in the couplings joining the barges.

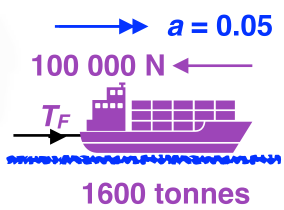
N.B. $400k$ N $= 400000$ N

Working: **The diagram for the question is below (the nearest to a barge I could find is a cargo ship — I hope you're not too upset by that).**
Since the driving force is coming from the barge at the back the directions of the tensions in the couplings are reversed. It wouldn't matter if you got them the wrong way round as the values would come out negative.
Let T_F be the tension in the coupling between the front 2 barges and T_B be the tension in the coupling between the back 2 barges.
Remember: 1 tonne $\equiv 1000$ kg

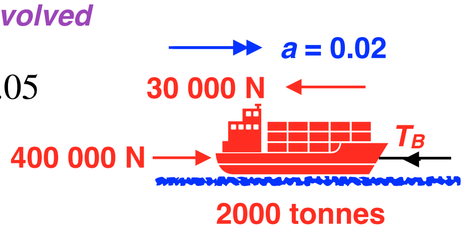


(a) ***Whole system so ignore the tensions in the couplings***
$F = ma(\rightarrow)$:
$400k - 100k - 200k - 300k = (2000k + 1400k + 1600k)a$
where k means 1000
$250 = 5000a \quad \Rightarrow \quad a = \frac{1}{20} = 0.05$m/s²
The acceleration of the barges is 0.05 m/s².

(b) ***Front barge POV, so T_B is not involved***
$F = ma(\rightarrow)$:
$T_F - 100k = 1600k \times 0.05$
where k means 1000
$T_F = 100k + 1600k \times 0.05$
$T_F = 180k$ N



Back barge POV, so T_F is not involved
$F = ma(\rightarrow)$:
$400k - T_B - 30k = 2000k \times 0.05$
where k means 1000
$T_B = 370k - 2000k \times 0.05$
$T_B = 270k$ N



The tension in the front coupling is $180k$ N and in the back coupling is $270k$ N.

E.g. 1 A light inextensible string passes over a smooth fixed pulley and carries particles of masses 5 kg and 7 kg, one at each end. If the system is moving freely, find in terms of g:

(a) the acceleration of each particle

(b) the tension in the string

(c) the force exerted on the pulley by the string.

Make sure you draw a diagram

Working

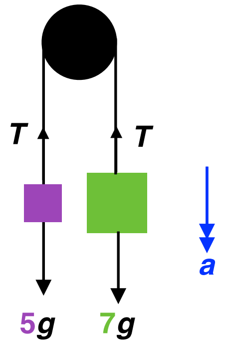
(a) $F = ma(\uparrow)$ 5 kg mass: $T - 5g = 5a$

$F = ma(\downarrow)$ 7 kg mass: $7g - T = 7a$

Add the equations to eliminate T: $2g = 12a$

Rearrange to get: $a = \frac{g}{6}$ m/s²

The acceleration of each particle is $\frac{g}{6}$ m/s².



(b) Replace a by $\frac{g}{6}$ in the 1st (or 2nd) equation

$T - 5g = 5 \times \frac{g}{6}$

$\therefore T = \frac{35g}{6}$ N$T = \frac{35g}{6}$

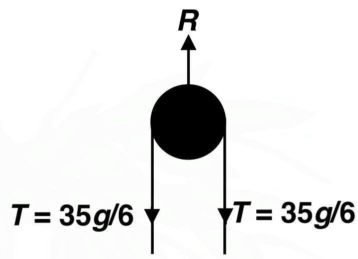
The tension in the string is $\frac{35g}{6}$ N.

(c) $R(\uparrow)$: $R = 2T$

$R = 2 \times \frac{35g}{6}$

$R = \frac{35g}{3}$

The force exerted on the pulley by the string is $\frac{35g}{3}$ N.



E.g. 2 A small block of mass 6 kg rests on a table top and is connected by a light inextensible string that passes over a smooth pulley, fixed on the edge of the table, to another small block of mass 5 kg which is hanging freely. Find, in terms of g, the acceleration of the system and the tension in the string if:

(a) the table is smooth

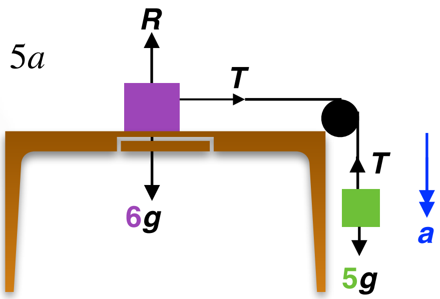
(b) the table is rough and exerts a frictional force of $2g$ N.

Make sure you draw a diagram.

Working (a)

$F = ma(\rightarrow)$ 6 kg mass: $T = 6a$

$F = ma(\downarrow)$ 5 kg mass: $5g - T = 5a$



Add the equations to eliminate T:

$$5g = 11a$$

$$a = \frac{5g}{11} \text{ m/s}^2$$

Replace a by $\frac{5g}{11}$ in the 1st (or 2nd) equation:

$$T = 6 \times \frac{5g}{11}$$

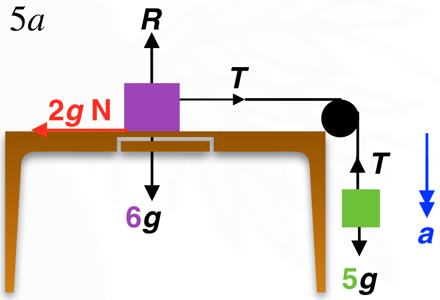
$$T = \frac{30g}{11} \text{ N}$$

The the acceleration of the system is $\frac{5g}{11}$ m/s² and the tension in the string is $\frac{30g}{11}$ N.

(b)

$F = ma(\rightarrow)$ 6 kg mass: $T - 2g = 6a$

$F = ma(\downarrow)$ 5 kg mass: $5g - T = 5a$



Add the equations to eliminate T:

$$3g = 11a$$

$$a = \frac{3g}{11} \text{ m/s}^2$$

Replace a by $\frac{3g}{11}$ in the 1st (or 2nd) equation:

$$T - 2g = 6 \times \frac{3g}{11}$$

$$T = \frac{40g}{11} \text{ N}$$

The the acceleration of the system is $\frac{3g}{11}$ m/s² and the tension in the string is $\frac{40g}{11}$ N.

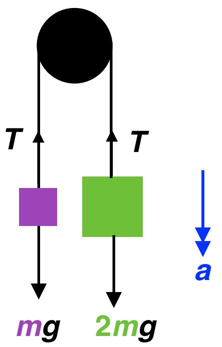
As you would expect, the frictional force has caused a decrease in the acceleration but an increase in the tension.

E.g. 3 Two particles P and Q are connected by a long inextensible string passing over a smooth pulley. The mass of P is m, the mass of Q is $2m$ and the particles are held so that each is 3 metres below the pulley. The system is released from rest and after 1 second the string breaks. Find:

(a) the speed of each particle at that instant

(b) the further distance that P rises.

Working (a) Consider what we know from a SUVAT perspective.
$u = 0, t = 1, v = ?$
To find the final speed, we need to calculate the acceleration of the system.



$F = ma(\uparrow)$ mass P: $T - mg = ma$

$F = ma(\downarrow)$ mass Q: $2mg - T = 2ma$

Add the equations to eliminate T: $mg = 3ma$

Rearrange to get: $a = \frac{g}{3}$ m/s²

SUVAT: $u = 0, t = 1, a = \frac{g}{3}, v = ?$

No s $\Rightarrow$ $v = u + at$: $v = 0 + \frac{g}{3} \times 1$

$$v = \frac{g}{3} = 3\frac{4}{15}$$

The speed of the particles when the string breaks is $3\frac{4}{15}$ m/s.

(b) ***After the string breaks, particle P moves under the influence of gravity so its acceleration is -9.8.***
Its initial velocity is the final velocity found in (a)
At its highest point, its velocity is 0.

SUVAT: $u = 3\frac{4}{15} = \frac{49}{15}, v = 0, a = -9.8, s = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $0^2 = \left(\frac{49}{15}\right)^2 + 2 \times (-9.8) \times s$

$$s = \frac{49}{90} \approx 0.544$$

The further distance that P rises is 0.544 m (3 s.f.)

Video: Pulley - vertical strings (1st video)
Video: Pulleys - vertical strings (continued motion - 2nd video)
Video: Pulley - edge of table
Video (force on pulley): Resultant force on a pulley

Pulley - vertical strings EQ
Pulley - edge of table EQ

Solutions to Starter and E.g.s

Exercise
p527 22E Qu 1-8