Converting Exponentials to a Linear Model

Starter

1. **(Review of last lesson)** A fungus is being grown under controlled conditions in a laboratory. Initially, it covers an area of 4 mm². After t hours, its are is N mm², where $N = N_0 e^{kt}$ and N_0 and k are constants. After 6 hours its area is 10 mm².
   (a) Find the values of N_0 and k.
   (b) Predict the area of the fungus after 12 hours.
   (c) How long will it take for the fungus to grow to 15 mm²?
   (d) Describe one limitation of the model used.

Working:

(a) When $t = 0$, the area is 4 mm²: so $N_0 = 4$

$N = 4e^{kt}$

When $t = 6, N = 10$:

$$10 = 4e^{6k}$$
$$2.5 = e^{6k}$$
$$\ln 2.5 = 6k$$
$$\frac{1}{6}\ln 2.5 = k$$
$$k = 0.153$$

So $N = 4e^{0.153t}$

(b) When $t = 12$, $N = 4e^{0.153\times 12} \approx 25$ mm².

(c) When $N = 15$,

$$15 = 4e^{0.153t}$$
$$3.75 = e^{0.153t}$$
$$\ln 3.75 = 0.153t$$
$$t = \frac{\ln 3.75}{0.153} \approx 8.66 \text{ hours}$$

Time taken to reach 15 mm² is 8h 39m

(d) There are no restrictions so according to the model the fungus would grow infinitely large.

t	0	1	2	3	4	5	6
N (measured)	4	5.2	5.9	6.9	7.8	9.1	10
$\ln N$	1.4	1.65	1.8	1.9	2.05	2.2	2.3

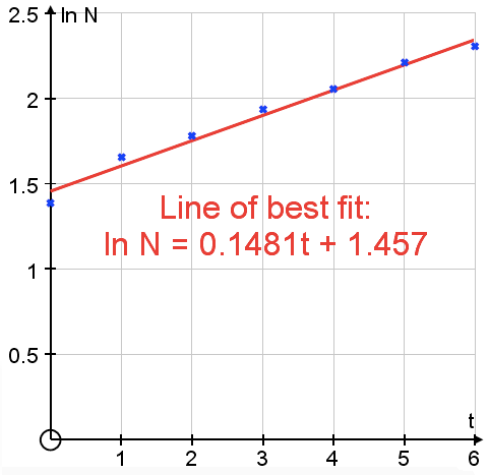
E.g. 1 (a) On graph paper, draw the graph of $\ln N$ ($y-$axis) against t ($x-$axis) and plot the points from the table.

(b) Draw a line of best fit through the points.
N.B. There should be roughly an equal number of points above and below the line.

(c) Calculate the gradient of the straight line and the note the $y-$intercept.
N.B. Each person's graph will be slightly different so the values of the gradient and $y-$intercept will be slightly different.

(d) Using your answers to (b), find the values of N_0 and k. Hence write down the exponential formula involving N and t.

Working: (a) and (b) See graph



(c) Gradient $\approx 0.15\ (\pm 0.02)$
$y-$intercept $\approx 1.46\ \pm 0.15$

(d) Gradient $= k$
So $k \approx 0.15\ (\pm 0.02)$

$y-$intercept = $\ln N_0$
so $N_0 = e^{\text{y-intercept}}$
$N_0 \approx e^{1.46}$
$N_0 \approx 4.3$
$(3.7 \leq N_0 \leq 5.0$ is ok)

Formula is $N = 4.3e^{0.15t}$ (compared to $N = 4e^{0.153t}$)

E.g. 2 Use your calculator to find the equation of the line of best fit for the data above.
N.B. Enter the $t-$values in the first column and $\ln N$ in the second column.

Working: $y = a + bx$ where $a = 1.45656$ and b = 0.1481

E.g. 3 The number of bacteria, p, in a petri dish is observed over a period of time, t. The bacteria population can be modelled over time by the formula $p = at^b$, where a and b are constants. The results from the observations are shown in the table below.

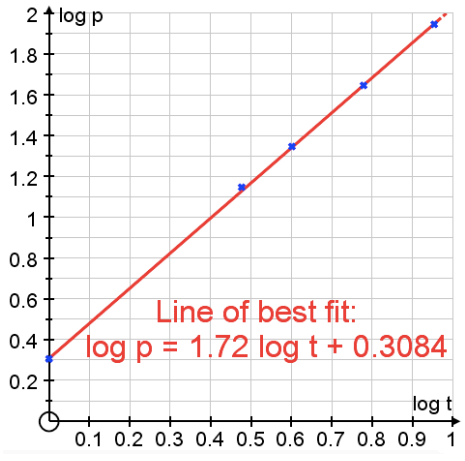
t (days)	1	3	4	6	9
p (1000s)	2	14	22	44	88

Plot a linear graph to represent this data and use this to find the values of a and b.

Working: $p = at^b$ leads to the linear relationship $\log p = b \log t + \log a$
Horizontal axis: $\log t$ Vertical axis: $\log p$

$\log t$	$\log 1 = 0$	$\log 3 = 0.48$	$\log 4 = 0.60$	$\log 6 = 0.78$	$\log 9 = 0.95$
$\log p$	$\log 2 = 0.30$	$\log 14 = 1.15$	$\log 22 = 1.34$	$\log 44 = 1.64$	$\log 88 = 1.94$

Plot the points on a graph
Draw a line of best fit through the points



Gradient $\approx 1.7\ \pm 0.2$
$y-$intercept $\approx 0.3\ \pm 0.03$

Linear relationship is:
$\log p = 1.7 \log t + 0.3$

$\log a = 0.3$ so $a = 10^{0.3} = 2.0$ (2 s.f.)
$b =$ gradient ≈ 1.7

$p = 2.0t^{1.7}$

E.g. 4 Two variable, S and x, satisfy the formula $S = 4 \times 7^x$.

(a) Find a linear relationship connecting S and x.

(b) The straight line graph of $\log S$ against x is plotted. Write down the gradient and the value of the intercept on the vertical axis.

Working:

(a) Take logs of both sides: $\log S = \log\left(4 \times 7^x\right)$

1st law of logs: $\log S = \log 4 + \log 7^x$

3rd law of logs $\log S = x \log 7 + \log 4$

(b) Gradient $= \log 7$

$y-$intercept $= \log 4$

Video: Modelling exponential curves - converting to linear form (3 videos)

Solutions to Starter and E.g.s

Exercise

p143 8D Qu 1, 2, 5, 6, 7 (no graph paper required)

Reducing exponentials to linear form 1 (Modelling) Qu 3-5 (need 1 sheet of A5 graph paper)