Definite integration

Starter

1. **(Review of last lesson)**

Given that $\dfrac{dy}{dx} = 2 + 3x^2$ and that $x = 2$ when $y = 9$, find y as a function of x.

Working: $y = \int (2 + 3x^2)dx = x^3 + 2x + c$

When $x = 2, y = 9$: $9 = 2^3 + 2 \times 2 + c \quad \Rightarrow \quad c = -3$

$\therefore y = x^3 + 2x - 3$

2. **(Review of last lesson)** A curve has a gradient function of $2x + k$, where k is a constant. It crosses the $y-$axis at $(0, -3)$ and the $x-$axis at $(1, 0)$ and $(a,\ 0)$. Find the equation of the curve and the value of a.

Working: $y = \int (2x + k)dx = x^2 + kx + c$

Substitute $(0, -3)$: $c = -3$

$y = x^2 + kx - 3$

Substitute $(1, 0)$: $0 = 1 + k - 3 \quad \Rightarrow \quad k = 2$

The equation of the curve is $y = x^2 + 2x - 3$;

Substitute $(a,\ 0)$: $0 = a^2 + 2a - 3 \quad \Rightarrow \quad (a + 3)(a - 1) = 0$

$a = -3$ or $a = 1$

Since we are given $(1, 0)$, $a = -3$.

E.g. 1 Find the exact value of:

(a) $\displaystyle\int_{-1}^{2} (6x^2 + 5)dx$ (b) $\displaystyle\int_{3}^{4} (x^2 + 3x)dx$

(c) $\displaystyle\int_{4}^{9} \sqrt{x}dx$ (d) $\displaystyle\int_{3}^{7} dx$

Working:

(a)
$$\int_{-1}^{2} (6x^2 + 5)dx = \left[2x^3 + 5x\right]_{-1}^{2}$$
$$= \left(2 \times 2^3 + 5 \times 2\right) - \left(2 \times (-1)^3 + 5 \times (-1)\right)$$
$$= 33$$

(b)
$$\int_{3}^{4} (x^2 + 3x)dx = \left[\frac{1}{3}x^3 + \frac{3}{2}x^2\right]_{3}^{4}$$
$$= \left(\frac{1}{3} \times 4^3 + \frac{3}{2} \times 4^2\right) - \left(\frac{1}{3} \times 3^3 + \frac{3}{2} \times 3^2\right)$$
$$= \frac{137}{6} = 22\frac{5}{6}$$

(c) $\int_4^9 \sqrt{x}dx = \int_4^9 x^{\frac{1}{2}}dx$

$$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$$
$$= \left[\frac{2}{3}(\sqrt{x})^3\right]_4^9$$
$$= \left(\frac{2}{3}\times(\sqrt{9})^3\right) - \left(\frac{2}{3}\times(\sqrt{4})^3\right)$$
$$= \left(\frac{2}{3}\times 27\right) - \left(\frac{2}{3}\times 8\right)$$
$$= \frac{38}{3} = 12\frac{2}{3}$$

(d) $\int_3^7 dx = \int_3^7 1dx = \Big[x\Big]_3^7 = 7 - 3 = 4$

E.g. 2 Given that $\int_0^a x^3 dx = 64$, find a, where $a > 0$.

Working: $\int_0^a x^3 dx = \left[\frac{1}{4}x^4\right]_0^a = \frac{1}{4}\times a^4 - \frac{1}{4}\times 0^4 = \frac{1}{4}a^4$

$\frac{1}{4}a^4 = 64 \quad \Rightarrow \quad a^4 = 256 \quad \therefore a = 4$

E.g. 3 Let $\int_a^b f(x)dx = 15$ and $\int_a^b g(x)dx = -7$. Find $\int_a^b (3f(x) - 4g(x))dx$.

Working: $\int_a^b (3f(x) - 4g(x))dx = \int_a^b 3f(x)dx - \int_a^b 4g(x)dx$

$$= 3\int_a^b f(x)dx - 4\int_a^b g(x)dx$$
$$= 3\times 15 - 4\times(-7)$$
$$= 45 + 28 = 73$$

E.g. 4 Find the possible values of A that satisfy $\int_2^3 (1-2Ax)dx = 6A^2$.

Working:

$$\int_2^3 (1-2Ax)dx = \left[x - Ax^2\right]_2^3$$
$$= \left(3 - A \times 3^2\right) - \left(2 - A \times 2^2\right)$$
$$= 3 - 9A - 2 + 4A$$
$$= 1 - 5A$$

So $6A^2 = 1 - 5A \quad \Rightarrow \quad 6A^2 + 5A - 1 = 0$

$$(6A - 1)(A + 1) = 0$$

$\therefore A = \frac{1}{6}$ or $A = -1$

Video: Definite integration

Definite integration EQ

Solutions to Starter and E.g.s

Exercise

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