Deriving the Constant Acceleration Formulae (single)

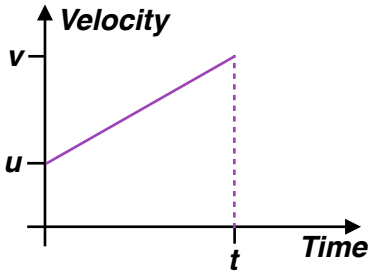
Starter

When a body moves with constant acceleration, the following letters are used:

s — displacement
u — initial velocity
v — final velocity
a — acceleration
t — time

Hence the following equations of motion are sometimes called the $SUVAT$ equations.

1. Consider the velocity-time graph.



(a) Using the fact that the gradient of a line is the acceleration, find an equation involving u, v, a and t.
Rearrange your equation to make v the subject.

(b) Using the fact that the area under the line is the displacement, find an equation involving s, u, v and t.

Working:

(a) Gradient $\equiv a = \dfrac{v-u}{t}$

Rearranging gives: $v = u + at$

(b) Area under the line $\equiv s = \dfrac{1}{2}(u+v)t$ ***area of trapezium***

Notes

From the starter there are 2 equations:

$v = u + at$ does not contain s

$s = \dfrac{1}{2}(u+v)t$ does not contain a

A further 3 equations can be derived by replacing a letter in $s = \dfrac{1}{2}(u+v)t$ by an expression from $v = u + at$.

Replacing v

Replace v by $u + at$ in $s = \dfrac{1}{2}(u+v)t$:

$$s = \frac{1}{2}(u+u+at)t$$
$$s = \frac{1}{2}(2ut + at^2)$$
$$s = ut + \frac{1}{2}at^2 \qquad \text{No } v$$

Replacing u

Rearrange $v = u + at$: $u = v - at$

Replace u by $v - at$ in $s = \dfrac{1}{2}(u+v)t$:

$$s = \frac{1}{2}(v-at+v)t$$
$$s = \frac{1}{2}(2vt - at^2)$$
$$s = vt - \frac{1}{2}at^2 \qquad \text{No } u$$

Replacing t

Rearrange $v = u + at$: $\quad t = \dfrac{v-u}{a}$

Replace t by $\dfrac{v-u}{a}$ in $s = \dfrac{1}{2}(u+v)t$: $\quad s = \dfrac{1}{2}\left(u+v\right)\left(\dfrac{v-u}{a}\right)$

$$s = \frac{1}{2a}\left(v^2 - u^2\right)$$

$$v^2 = u^2 + 2as$$ No t

Video: Deriving the constant acceleration (SUVAT) equations

Solutions to Starter and E.g.s

No Exercise

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