Describing Vectors

Starter

1. **(Review of GCSE material)** State the position of the point $(2, -3)$ after being translated under the following vectors:

(a) $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ (b) $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$

Working: (a) $(2+5, -3+-6) = (7, -9)$

(b) $(2+(-3), -3+7) = (-1, 4)$

2. **(Review of GCSE material)** Find the magnitude of the vector $\begin{pmatrix} 7 \\ -24 \end{pmatrix}$.

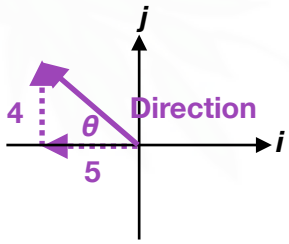
Working: Magnitude $= \sqrt{7^2 + (-24)^2} = 25$

E.g. 1 Write the vector $(5, 60^\circ)$ in component form, using $\mathbf{i}$ and $\mathbf{j}$ notation.

Working: $(5, 60^\circ) \equiv \begin{pmatrix} 5\cos 60^\circ \\ 5\sin 60^\circ \end{pmatrix} = \begin{pmatrix} 2.5 \\ \frac{5\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix} = 2.5\mathbf{i} + 4.33\mathbf{j}$

E.g. 2 Write $-5\mathbf{i} + 4\mathbf{j}$ in magnitude-direction form.

Working: $|-5\mathbf{i} + 4\mathbf{j}| = \sqrt{(-5)^2 + 4^2} = \sqrt{41}$



$-5\mathbf{i} + 4\mathbf{j}$ is in the 2nd quadrant

$\theta = \tan^{-1}\frac{4}{5} = 38.66^\circ$

So direction is $180^\circ - 38.66^\circ = 141.3^\circ$

N.B. Direction measured anti-clockwise from positive $x-$axis

$-5\mathbf{i} + 4\mathbf{j} \equiv \left(\sqrt{41},\ 141.3^\circ\right)$

E.g. 3 Given that the vectors $-3\mathbf{i} + 8\mathbf{j}$ and $6\mathbf{i} + x\mathbf{j}$ are parallel, find the value of x.

Working: -3 to 6 is $\times(-2)$

So $x = 8 \times (-2) = -16$

E.g. 4 Let $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$. Find the unit vector in the direction of $\mathbf{v}$.

Working: $|\mathbf{v}| = |3\mathbf{i} + 4\mathbf{j}| = \sqrt{3^2 + 4^2} = 5$

So $\hat{\mathbf{v}} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$

E.g. 5 Let $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$. Find:

(a) the unit vector and

(b) the vector of length 26 units in the direction of $\mathbf{v}$.

Working: (a) $\hat{\mathbf{v}} = \dfrac{5\mathbf{i} - 12\mathbf{j}}{\sqrt{5^2 + (-12)^2}} = \dfrac{1}{13}(5\mathbf{i} - 12\mathbf{j})$

(b) Vector of length 26 units = $26 \times \dfrac{1}{13}(5\mathbf{i} - 12\mathbf{j}) = 2(5\mathbf{i} - 12\mathbf{j})$

Video: What is a vector and scalar quantity?
Video: Magnitude of a vector
Video: Magnitude and direction of a vector
Video: Unit vectors

Solutions to Starter and E.g.s

Exercise
p225 12A Qu 2i, 3i, 4-9