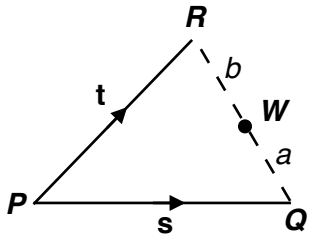
Differentiating from First Principles

Starter

1. **(Review of last lesson)** $\overrightarrow{PQ} = \mathbf{s}$ and $\overrightarrow{PR} = \mathbf{t}$. The point W lies on QR and divides it in the ratio $a : b$. Given that $\overrightarrow{PW} = \frac{5}{9}\mathbf{s} + \frac{4}{9}\mathbf{t}$, find the values a and b.



Working:

$$\overrightarrow{QR} = \mathbf{t} - \mathbf{s}$$

$$\overrightarrow{QW} = \frac{a}{a+b}\overrightarrow{QR} = \frac{a}{a+b}(\mathbf{t} - \mathbf{s})$$

$$\overrightarrow{PW} = \overrightarrow{PQ} + \overrightarrow{QW} = \mathbf{s} + \frac{a}{a+b}(\mathbf{t} - \mathbf{s}) \equiv \frac{5}{9}\mathbf{s} + \frac{4}{9}\mathbf{t}$$

Equating coefficients of $\mathbf{t}$: $\frac{a}{a+b} = \frac{4}{9}$

By inspection, $a = 4$, $b = 5$

E.g. 1 Differentiate the function $f(x) = x^2 - 7x$ from first principles.

Working:

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{x + \delta x - x}$$

$$f'(x) = \lim_{\delta x \to 0} \frac{(x + \delta x)^2 + 7(x + \delta x) - (x^2 + 7x)}{x + \delta x - x}$$ ***replace*** x ***by*** $x + \delta x$

$$f'(x) = \lim_{\delta x \to 0} \frac{x^2 + 2x\delta x + (\delta x)^2 + 7x + 7\delta x - x^2 - 7x}{\delta x}$$ ***expand***

$$f'(x) = \lim_{\delta x \to 0} \frac{2x\delta x + (\delta x)^2 + 7\delta x}{\delta x}$$ ***original function*** $x^2 - 7x$ ***disappears***

$$f'(x) = \lim_{\delta x \to 0} \frac{\delta x(2x + \delta x + 7)}{\delta x}$$ ***factorise*** δx ***out***

$$f'(x) = \lim_{\delta x \to 0} (2x + \delta x + 7)$$ ***cancel*** δx ***in the numerator and denominator***

$$f'(x) = 2x - 7$$ ***remove*** $\lim_{\delta x \to 0}$ ***and*** δx

E.g. 2 Differentiate the function $f(x) = 4x^3$ from first principles.

Working:

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{x + \delta x - x}$$

$$f'(x) = \lim_{\delta x \to 0} \frac{4(x+\delta x)^3 - 4x^3}{x + \delta x - x}$$ ***replace*** x ***by*** $x + \delta x$

$$f'(x) = \lim_{\delta x \to 0} \frac{4\left(x^3 + 3x^2\delta x + 3x(\delta x)^2 + (\delta x)^3\right) - 4x^3}{\delta x}$$ ***expand***

$$f'(x) = \lim_{\delta x \to 0} \frac{4x^3 + 12x^2\delta x + 12x(\delta x)^2 + 3(\delta x)^3 - 4x^3}{\delta x}$$ ***expand***

$$f'(x) = \lim_{\delta x \to 0} \frac{12x^2\delta x + 12x(\delta x)^2 + 3(\delta x)^3}{\delta x}$$ ***original function*** $4x^3$ ***disappears***

$$f'(x) = \lim_{\delta x \to 0} \frac{\delta x\left(12x^2 + 12x\delta x + 3(\delta x)^2\right)}{\delta x}$$ ***factorise*** δx ***out***

$$f'(x) = \lim_{\delta x \to 0} 12x^2 + 12x\delta x + 3(\delta x)^2$$ ***cancel*** δx ***in numerator and denominator***

$$f'(x) = 12x^2$$ ***remove*** $\lim_{\delta x \to 0}$ ***and*** δx

Video: Differentiation from 1st principles (2nd video)

Solutions to Starter and E.g.s

Exercise
p253 13B Qu 3, 5, 8, 9