# Discriminant

**Starter**

1. (Review of last lesson) Find the set of values that satisfies $3x + 5 < 17$ and $x^2 - 7x + 10 < 0$.

***Working:*** Linear: $3x < 12$ so $x < 4$

Quadratic: Solve: $x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

Roots are $x = 2$ and $x = 5$

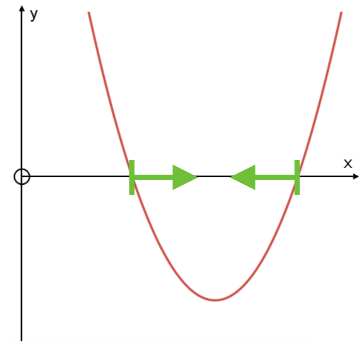
Coefficient of $x^2$ is +ve so concave-up

$< 0 \Rightarrow$ below the $x-$axis

We need the $x-$values to the right of 2 and to the left of 5.

$2 \le x \le 5$

Overall $\{x : x > 2\} \cap \{x : x < 4\}$




2. (Review of last lesson) Solve the inequality $6 + \dfrac{5}{x} \ge \dfrac{8}{x}$.

***Working:*** Multiplying by $x^2$: $6x^2 + 5x \ge 8x$

Rearranging: $6x^2 - 3x \ge 0$

Solving: $3x(2x-1) \ge 0$

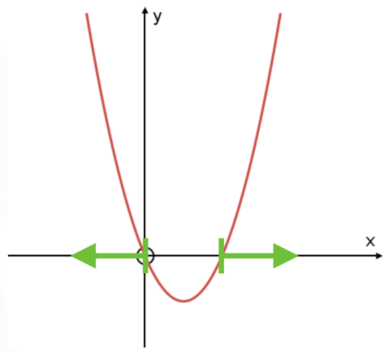
$\therefore$ the roots are $x = 0$ or $x = \dfrac{1}{2}$

Coefficient of $x^2$ is +ve so concave-up

$\ge 0 \Rightarrow$ above the $x-$axis

We need the $x-$values to the left of 0 and to the right of 0.5.

$\{x : x \le 0\} \cup \{x : x \ge 0.5\}$




3. How many roots can a quadratic equation have? Draw diagrams to illustrate your answer.

***Working:*** See notes.

***E.g. 1*** Without solving the equation, determine the nature of the roots:

(a) $x^2 - 6x + 4 = 0$ (b) $3x^2 + 4x + 2 = 0$

***Working:***

(a) $a = 1$ $b = -6$ $c = 4$

$b^2 - 4ac = (-6)^2 - 4 \times 1 \times 4 > 0$ *(exact value not needed)*

Since discriminant > 0, 2 real roots

(b) $a = 3, b = 4, c = 2$

$b^2 - 4ac = 4^2 - 4 \times 3 \times 2 < 0$ *(exact value not needed)*

Since discriminant < 0, no real roots

***E.g. 2*** If the roots of $3x^2 + kx + 12 = 0$ are equal, find $k$.

***Working:*** Equal roots $\Rightarrow$ $b^2 - 4ac = 0$

$a = 3$ $b = k$ $c = 12$

$k^2 - 4 \times 3 \times 12 = 0$

$k^2 - 144 = 0$

$k = \pm 12$

***E.g. 3*** Find the range of values of $k$ for which $x^2 - kx + 4 = 0$ has two real and distinct roots.

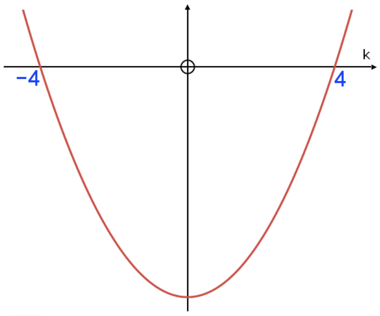
***Working:*** Two real and distinct roots $\Rightarrow$ $b^2 - 4ac > 0$

$a = 1, b = -k, c = 4$

$(-k)^2 - 4 \times 1 \times 4 > 0$

$\Rightarrow$ $k^2 - 16 > 0$

Roots are $k = -4$ and $k = 4$




Coefficient of $k^2$ is +ve so concave-up

$> 0 \Rightarrow$ above the $x-$axis

We need the $k-$values **to the left of −4** and **to the right of 4**.

$\{k : k < -4\} \cup \{k : k > 4\}$

***E.g. 4*** Find the range of values of k for which the curve $y = 2x^2 - (k+1)x + 2$ has no real roots.

***Working:*** No real roots $\Rightarrow$ $b^2 - 4ac < 0$

$a = 2$ $b = -(k+1)$ $c = 2$

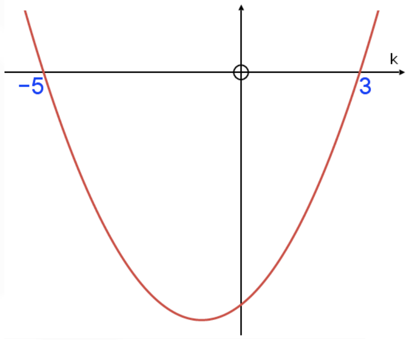
$(-(k+1))^2 - 4 \times 2 \times 2 < 0$

$\Rightarrow$ $k^2 + 2k + 1 - 16 > 0$

$\therefore k^2 + 2k - 15 > 0$

Solving $k^2 + 2k - 15 = 0$

gives roots as $k = -5$ and $k = 3$




Coefficient of $k^2$ is +ve so concave-up

$< 0 \Rightarrow$ below the $x-$axis

We need the $k-$values **to the right of −5** and **to the left of 3**.

$\{k : k > -5\} \cap \{k : k < 3\}$

***E.g. 5*** The height of a ball, $h$ m, at time $t$ seconds is given by $h = 6 + 20t - 4.9t^2$. Prove that the ball does not reach a height of 32 m.

***Working:***

$32 = 6 + 20t - 4.9t^2$
$4.9t^2 - 20t + 26 = 0$
$a = 4.9 \qquad b = -20 \qquad c = 26$
$(-20)^2 - 4 \times 4.9 \times 26 < 0$
Since $b^2 - 4ac < 0$, there are no solutions to the equation
$32 = 6 + 20t - 4.9t^2$ so the ball does not reach a height of 32 m.

Video: Discriminant

Solutions to Starter and E.g.s

**Exercise**
p48 3E Qu 1i, 2i, 3i, 4-10