Disguised Quadratics

Starter

1. **(Review of last lesson)** For what values of k does the equation $x^2 + (3k-1)x + 10 + 2k = 0$ have real roots.

Working:

Real roots $\Rightarrow$ $b^2 - 4ac \geq 0$

$a = 1 \quad b = 3k - 1 \quad c = 10 + 2k$

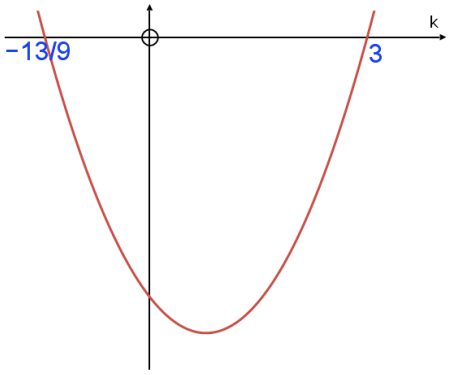
$(3k-1)^2 - 4 \times 1 \times (10 + 2k) \geq 0$

$9k^2 - 6k + 1 - 40 - 8k \geq 0$

$9k^2 - 14k - 39 \geq 0$

Solving $9k^2 - 14k - 39 = 0$

gives roots as $k = -\frac{13}{9}$ and $k = 3$



Coefficient of k^2 is +ve so concave-up

$\geq 0 \Rightarrow$ above the $x-$axis

We need the $k-$values **to the left of** $-\frac{13}{9}$ and **to the right of 3**.

$$\left\{k : k \leq -\frac{13}{9}\right\} \cup \left\{k : k \geq 3\right\}$$

2. Solve the equations:

(a) $u^2 - 5u - 36 = 0$ (b) $y^4 - 5y^2 - 36 = 0$ (c) $a - 5\sqrt{a} - 36 = 0$

Working:

(a) $(u-9)(u+4) = 0$

$u = 9$ or $u = -4$

(b) See below.

(c) See below.

E.g. 1 Solve:

(a) $x - 2\sqrt{x} - 3 = 0$ (b) $x^4 - 24x^2 - 25 = 0$ (c) $p^6 - 7p^3 - 8 = 0$

Working:

(a) Let $u = \sqrt{x}$ to get $u^2 - 2u - 3 = 0$

Solving gives $(u-3)(u+1) = 0$

$u = 3$ or $u = -1$

$\sqrt{x} = 3$ or $\sqrt{x} = -1$

$x = 9$ or No solution

(b) Let $u = x^2$ to get $u^2 - 24u - 25 = 0$

Solving gives $(u-25)(u+1) = 0$

$u = 25$ or $u = -1$

$x^2 = 25$ or $x^2 = -1$

$x = \pm 5$ or No real solution

(c) Let $u = p^3$ to get $u^2 - 7u - 8 = 0$

Solving gives $(u-8)(u+1)=0$

$u=8$ or $u=-1$

$p^3=8$ or $p^3=-1$

$p=2$ or $p=-1$

E.g. 2 Solve $x+1=\dfrac{6}{x}$.

Working: Multiply by x and expand: $x^2+x=6$

$\therefore x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ or $x=2$

E.g. 3 Solve $x-2=\dfrac{4}{x+1}$

Working: Cross multiply: $(x-2)(x+1)=4$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3$ or $x=-2$

E.g. 4 Solve $2^{2x}-9\times 2^x+8=0$.

Hint: let $u=2^x$

Working: $u^2-9u+8=0$

$(u-8)(u-1)=0$

$u=8$ or $u=1$

$2^x=8$ or $2^x=1$

$x=3$ or $x=0$

E.g. 5 $2^{2x}-2^{x+1}+1=0$

Hint: rewrite 2^{x+1} as 2×2^x

Working: $u^2-2u+1=0$

$(u-1)(u-1)=0$

$u=1$

$2^x=1$

$x=0$

Video: Disguised quadratics

Solutions to Starter and E.g.s

Exercise

p50 3F Qu 1i, 2-8