# Disguised Quadratics using Logs

**Starter**

1. **(Review of last lesson)** Solve: (a) $5^x = 2^{2x+1}$ (b) $8 \times 5^{x-3} = 7 \times 9^x$

Give your answers exactly (i.e. in terms of logarithms).

***Working:*** (a)

Take logs of both sides: $\log 5^x = \log 2^{2x+1}$

3rd law of logs: $x \log 5 = (2x+1)\log 2$

Expand the brackets: $x \log 5 = 2x \log 2 + \log 2$

Collect like terms: $x \log 5 - x \log 4 = \log 2$

Factorise: $x(\log 5 - \log 4) = \log 2$

$$x = \frac{\log 2}{\log 5 - \log 4} = \frac{\log 2}{\log \frac{5}{4}}$$

***N.B.*** $x = 3.11$

(b)

Take logs of both sides: $\log\left(8 \times 5^{x-3}\right) = \log\left(7 \times 9^x\right)$

1st law of logs: $\log 8 + \log 5^{x-3} = \log 7 + \log 9^x$

3rd law of logs: $\log 8 + (x-3)\log 5 = \log 7 + x \log 9$

Expand: $\log 8 + x \log 5 - 3\log 5 = \log 7 + x \log 9$

Collect like terms: $x \log 5 - x \log 9 = \log 7 + 3\log 5 - \log 8$

Factorise: $x(\log 5 - \log 9) = \log 7 + 3\log 5 - \log 8$

Exact answer: $$x = \frac{\log 7 + 3\log 5 - \log 8}{\log 5 - \log 9} = \frac{\log \frac{875}{8}}{\log \frac{5}{9}}$$

***N.B.*** $x = -7.99$

2. Solve $5^{2x} - 12(5^x) + 20 = 0$ giving your answers to 3 s.f.

***Hint:*** Let $u = 5^x$.

***Working:*** Let $u = 5^x$ $\Rightarrow$

$$u^2 - 12u + 20 = 0$$
$$(u-10)(u-2) = 0$$

$u = 10$ or $u = 2$

$5^x = 10$ or $5^x = 2$

Take logs of both sides: $\log 5^x = \log 10$ or $\log 5^x = \log 2$

3rd law: $x \log 5 = \log 10$ or $x \log 5 = \log 2$

Exact answers: $x = \dfrac{\log 10}{\log 5}$ or $x = \dfrac{\log 2}{\log 5}$

To 3 s.f.: $x = 1.43$ or $x = 0.431$

***E.g. 1*** Solve $3^{2x} - 15(3^x) + 44 = 0$ giving your answers to 3 s.f.

***Working:*** Let $u = 3^x$ $\Rightarrow$

$$u^2 - 15u + 44 = 0$$
$$(u-11)(u-4) = 0$$

$u = 11$ or $u = 4$

$3^x = 11$ or $3^x = 4$

Take logs of both sides: $\log 3^x = \log 11$ or $\log 3^x = \log 4$

3rd law: $x \log 3 = \log 11$ or $x \log 3 = \log 4$

Exact answers: $x = \dfrac{\log 11}{\log 3}$ or $x = \dfrac{\log 4}{\log 3}$

To 3 s.f.: $x = 1.26$ or $x = 2.18$

***E.g. 2*** Solve $3^{2x} + 3^{x+1} - 10 = 0$ giving your answers to 3 s.f.

***Working:***

$3^{2x} + 3^{x+1} - 10 = 0 \Rightarrow 3^{2x} + 3 \times 3^x - 10 = 0$

Let $u = 3^x \Rightarrow u^2 + 3u - 10 = 0$

$(u-2)(u+5) = 0$

$u = 2$ or $u = -5$

$3^x = 2$ or $3^x = -5$

Take logs of both sides: $\log 3^x = \log 2$ or No solution

3rd law: $x \log 3 = \log 2$

Exact answers: $x = \dfrac{\log 2}{\log 3}$

To 3 s.f.: $x = 0.631$

***Remember*** For $\log x$ to exist $x > 0$

***E.g. 2*** Solve $3^{2x} + 3^{x+1} - 10 = 0$ giving your answers to 3 s.f.

***Working:***

$3^{2x} + 3^{x+1} - 10 = 0 \Rightarrow 3^{2x} + 3 \times 3^x - 10 = 0$

Let $u = 3^x \Rightarrow u^2 + 3u - 10 = 0$

$(u-2)(u+5) = 0$

$u = 2$ or $u = -5$

$3^x = 2$ or $3^x = -5$

Take logs of both sides: $\log 3^x = \log 2$ or No solution

3rd law: $x \log 3 = \log 2$

Exact answers: $x = \dfrac{\log 2}{\log 3}$

To 3 s.f.: $x = 0.631$

***Remember*** For $\log x$ to exist $x > 0$

***E.g. 3*** Solve $7^{2x} + 12 = 7^{x+1}$ giving your answers to 3 s.f.

***Working:***

$7^{2x} + 12 = 7^{x+1} \Rightarrow 7^{2x} - 7 \times 7^x + 12 = 0$

Let $u = 7^x \Rightarrow u^2 - 7u + 12 = 0$

$(u-4)(u-3) = 0$

$u = 4$ or $u = 3$

$7^x = 4$ or $7^x = 3$

Take logs of both sides: $\log 7^x = \log 4$ or $\log 7^x = \log 3$

3rd law: $x \log 7 = \log 4$ or $x \log 7 = \log 3$

Exact answers: $x = \dfrac{\log 4}{\log 7}$ or $x = \dfrac{\log 3}{\log 7}$

To 3 s.f.: $x = 0.712$ or $x = 0.565$

Video: Disguised quadratics involving exponentials

Solutions to Starter and E.g.s

**Exercise**

p120 7D Qu 1ia-d, 2-6