

Displacement, Velocity and Acceleration (single)

Starter

1. (Review of last lesson)

Find the total area between the curve $y = x(x^2 - 1)$ and the x -axis.

Working: Solve $x(x^2 - 1) = 0$ to find the roots: $y = x(x - 1)(x + 1)$
So the roots are $x = -1, x = 0$ and $x = 1$

The two areas are $\int_{-1}^0 x(x^2 - 1)dx$ and $\int_0^1 x(x^2 - 1)dx$

One of them will come out negative so it needs to be made positive before adding the areas.

$$\begin{aligned}\int_0^1 x(x^2 - 1)dx &= \int_0^1 (x^3 - x)dx \\ &= \left[\frac{1}{4}x^4 - \frac{1}{2}x^2 \right]_0^1 \\ &= \left(\frac{1}{4} - \frac{1}{2} \right) - (0 - 0) = -\frac{1}{4}\end{aligned}$$

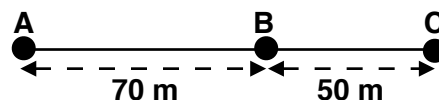
$$\begin{aligned}\int_{-1}^0 x(x^2 - 1)dx &= \int_{-1}^0 (x^3 - x)dx \\ &= \left[\frac{1}{4}x^4 - \frac{1}{2}x^2 \right]_{-1}^0 \\ &= (0 - 0) - \left(\frac{1}{4} - \frac{1}{2} \right) = \frac{1}{4}\end{aligned}$$

$$\text{The total area} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

E.g. 1 A particle starts at A and takes 8 seconds to travel to B and then another 4 seconds to go from B to C . After reaching C it completes its journey by returning to B in 3 seconds.

(a) For the journey from A to C find:

- (i) the average speed
(ii) the average velocity.



(b) For the **complete** journey from A to B find:

- (i) the average speed
(ii) the average velocity.

N.B. The complete journey means A to B to C to B

Working:

(a) (i) Distance = $70 + 50 = 120$ m
Time from A to C = $8 + 4 = 12$ s
Average speed = $\frac{120}{12} = 10$ m/s

(ii) *The distance and displacement are the same for the journey from A to C*

Displacement = $70 + 50 = 120$ m
Time from A to C = $8 + 4 = 12$ s
Average velocity = $\frac{120}{12} = 10$ m/s

(b) (i) Distance = $70 + 50 + 50 = 170$ m
Time from A to B = $8 + 4 + 3 = 15$ s
Average speed = $\frac{170}{15} = 11\frac{1}{3}$ m/s

(ii) Displacement = $70 + 50 - 50 = 70$ m

N.B. Displacement is a vector so direction is important.

That is the displacement from C to B is -50 m.

Time from A to B = $8 + 4 + 3 = 15$ s
Average velocity = $\frac{70}{15} = 4\frac{2}{3}$ m/s

Video: [Displacement](#)

Video: [Converting lengths and speeds on the Classwiz](#)

[Solutions to Starter and E.g.s](#)

Exercise

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