Equations of Normals to Curves

Starter

For ***perpendicular lines***, the product of the gradients is -1 i.e. $m_1 \times m_2 = -1$

1. **(Review of GCSE material)** Fill in the table with the perpendicular gradients.

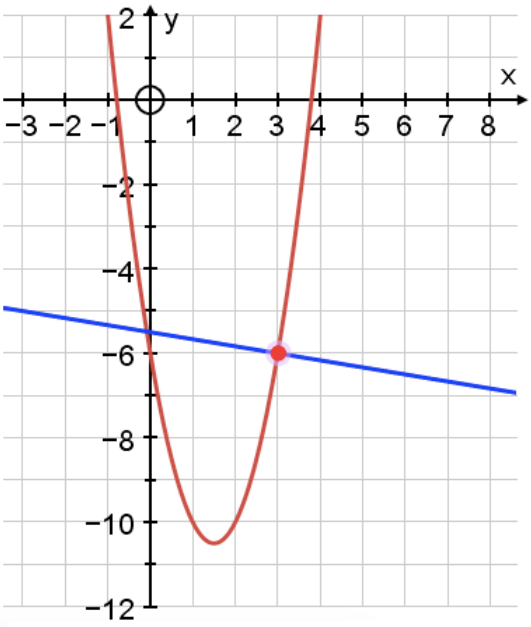
Gradient	Perpendicular gradient
5	
-7	
$\frac{1}{8}$	
$-\frac{2}{3}$	
$-3\frac{4}{7}$	
4.6	

Working:

Gradient	Perpendicular gradient
5	$-\frac{1}{5}$
-7	$\frac{1}{7}$
$\frac{1}{8}$	8
$-\frac{2}{3}$	$\frac{3}{2}$
$-3\frac{4}{7} = -\frac{25}{7}$	$\frac{7}{25}$
$4.6 = \frac{46}{10} = \frac{23}{5}$	$-\frac{5}{23}$

N.B. When a line is ***normal to a curve***, it means that is is ***perpendicular*** to it at the the point where they intersect.

2. Find the equation of the ***normal*** to the curve $y = 2x^2 - 6x - 6$ at the point $(3, -6)$. Express your answer in the form $ax + by + c = 0$, where a, b and c are integers.



Working: $\frac{dy}{dx} = 4x - 6$

When $x = 3$,

$\frac{dy}{dx} = 4 \times 3 - 6 = 6$ ***gradient of tangent***

So gradient of normal is $-\frac{1}{6}$ ***negative reciprocal***

So gradient is $-\frac{1}{2}$, passes through $(3, -6)$

Using $y - y_1 = m(x - x_1)$: $y + 6 = -\frac{1}{6}(x - 3)$

$y = -\frac{1}{6}x - \frac{11}{2}$

Multiply by 6 and rearrange: $x + 6y + 33 = 0$

E.g. 1 Find the equation of the normal to the curve $y = x^3 - 4x$ when $x = 3$. Express your answer in the form $ax + by = c$ where a, b and c are integers.

Working: $\frac{dy}{dx} = 3x^2 - 4$

When $x = 3$, $\frac{dy}{dx} = 3 \times 3^2 - 4 = 23$ ***gradient of tangent***

So gradient of normal is $-\frac{1}{23}$ ***negative reciprocal***

When $x = 3$, $y = 3^3 - 4 \times 3 = 15$

So gradient is $-\frac{1}{23}$, passes through $(3, 15)$

Using $y - y_1 = m(x - x_1)$: $y - 15 = -\frac{1}{23}(x - 3)$

Multiply by 23 and rearrange: $x + 23y = 348$

E.g. 2 Find the equation of the normal to the curve $y = x^2 - 4x - 3$ at the point where the curve cuts the $y-$axis. Express your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Working: The curve intersects the $y-$axis when $x = 0 \quad \Rightarrow \quad y = -3$

$\frac{dy}{dx} = 2x - 4$

When $x = 0$, $\frac{dy}{dx} = 2 \times 0 - 4 = -4$ ***gradient of tangent***

So gradient of normal is $\frac{1}{4}$ ***negative reciprocal***

So gradient is $\frac{1}{4}$, passes through (0, -3)

Using $y - y_1 = m(x - x_1)$: $y + 3 = \frac{1}{4}(x - 0)$

Multiply by 4: $4y + 12 = x$

$x - 4y - 12 = 0$

Video: Equation of tangents/normals

Solutions to Starter and E.g.s

Exercise

Normal: p274 14A Qu 1i, 3, 4, 8, 10, 13 (not 12)