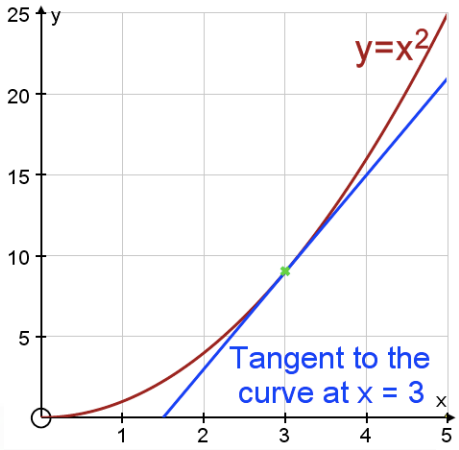
Equations of Tangents to Curves

Starter

1. **(Review of GCSE material)** Find the equation of the straight line which has a gradient of 2 and passes through the point $(3, 8)$.

Working: Using $y - y_1 = m(x - x_1)$: $y - 8 = 2(x - 3)$
$y = 2x + 2$

2. Find the equation of the tangent to the curve $y = x^2$ at the point where $x = 3$.



Working:
$\frac{dy}{dx} = 2x$
When $x = 3$, $\frac{dy}{dx} = 6$
When $x = 3$, $y = 3^2 = 9$
So gradient is 6, passes through (3, 9)
Using $y - y_1 = m(x - x_1)$
$y - 9 = 6(x - 3)$
$y = 6x - 9$

E.g. 1 Find the equation of the tangent to the curve $y = x^2 - 3x + 2$ at the point where $x = 3$.

Working:
$\frac{dy}{dx} = 2x - 3$
When $x = 3$, $\frac{dy}{dx} = 6 - 3 = 3$
When $x = 3$, $y = 3^2 - 3 \times 3 + 2 = 2$
So gradient is 3, passes through $(3, 2)$
Using $y - y_1 = m(x - x_1)$: $y - 2 = 3(x - 3)$
$y = 3x - 7$

E.g. 2 Find the equation of the tangent to the curve $y = x^2 + x - 4$ which has gradient -5.

Working: ***We are given the gradient so we need to find the point at which curve has gradient -5.***
$\frac{dy}{dx} = 2x + 1$
Gradient is -5 when, $2x + 1 = -5$ $\Rightarrow$ $x = -3$
When $x = -3$, $y = (-3)^2 + (-3) - 4 = 2$
So gradient is -5, passes through $(-3, 2)$
Using $y - y_1 = m(x - x_1)$: $y - 2 = -5(x + 3)$
$y = -5x - 13$

E.g. 3 Find the value of k for which $y = 2x + k$ is a tangent to the curve $y = 2x^2 - 3$.

Working:

Gradient method

$y = 2x + k$ has gradient 2

So $y = 2x^2 - 3$ must also have gradient 2 — we need to find the point at which this occurs

$\frac{dy}{dx} = 4x$

$\frac{dy}{dx} = 2$ when $4x = 2 \quad \Rightarrow \quad x = \frac{1}{2}$

When $x = \frac{1}{2}$, $y = 2 \times \left(\frac{1}{2}\right)^2 - 3 = -2.5 = -\frac{5}{2}$

Substitute into $y = 2x + k$: $\quad -\frac{5}{2} = 2 \times \frac{1}{2} + k$

$\therefore k = -\frac{7}{2} = -3.5$

Discriminant method

The line intersects the curve when $2x^2 - 3 = 2x + k$

i.e. $2x^2 - 2x - 3 - k = 0$

The line is tangent to the curve when there is only 1 root i.e. $b^2 - 4ac = 0$

$a = 2 \quad b = -2 \quad c = -3 - k$

$(-2)^2 - 4 \times 2 \times (-3 - k) = 0 \quad \Rightarrow \quad 4 + 8(3 + k) = 0$

$8k = -28 \quad \therefore k = -\frac{7}{2} = -3.5$

Video: Equation of tangents/normals

Solutions to Starter and E.g.s

Exercise

Tangent: p274 14A Qu 1i (tangent), 2, 5, 6, 9, 11