Finding the Constant of Integration

Starter

1. (Review of last lesson)

Find: (a) $\int -\frac{5}{9x^8} dx$ (b) $\int \left(\frac{2x^3 - \sqrt{x}}{x}\right) dx$

Working: (a) $\int -\frac{5}{9x^8} dx = \int -\frac{5x^{-8}}{9} dx = -\frac{5x^{-7}}{9 \times (-7)} + c = \frac{5}{72x^7} + c$

(b) $$\int \left(\frac{2x^3 - \sqrt{x}}{x}\right) dx = \int \left(\frac{2x^3}{x} - \frac{x^{\frac{1}{2}}}{x}\right) dx$$
$$= \int \left(2x^2 - x^{-\frac{1}{2}}\right) dx$$
$$= \frac{2}{3}x^3 - 2x^{\frac{1}{2}} + c$$
$$= \frac{2}{3}x^3 - 2\sqrt{x} + c$$

2. A curve passes through the point $(2, -5)$ and satisfies $\frac{dy}{dx} = 6x^2 - 1$. Find y in terms of x.

Working: $y = \int (6x^2 - 1) dx = 2x^3 - x + c$

Substituting $(2, -5)$: $-5 = 2 \times 2^3 - 2 + c$

$\therefore c = -19$

$y = 2x^3 - x - 19$

E.g. 1 The gradient function of a curve is $4x$ and it passes through $(2, 11)$. Find the equation of the curve.

Working: $y = \int 4x dx = 2x^2 + c$

Substitute $(2, 11)$: $11 = 2 \times 2^2 + c \Rightarrow c = 3$

The equation of the curve is $y = 2x^2 + 3$.

E.g. 2 A tree is growing so that, after t years, its height is increasing at a rate of $\frac{30}{\sqrt[3]{t}}$ cm per year. Assume that, when $t = 0$, the height is 5 cm.
(a) Find the height of the tree to the nearest centimetre after 4 years.
(b) After how many years will the height be 4.1 metres?

Working: (a) Let h be the height of the tree

$$h = \int \frac{30}{\sqrt[3]{t}} dt = \int 30t^{-\frac{1}{3}} dt = 45t^{\frac{2}{3}} + c = 45\sqrt[3]{t^2} + c$$

When $t = 0, h = 5$: $c = 5$

$$h = 45\sqrt[3]{t^2} + 5$$

When $t = 4$: $h = 45\sqrt[3]{4^2} + 5 = 118.4$ (4 s.f.)

The height of the tree after 4 years is 118 cm (nearest cm)

(b) When $h = 410$:

$$45\sqrt[3]{t^2} + 5 = 410$$
$$\sqrt[3]{t^2} = 9$$
$$\left(\sqrt[3]{t}\right)^2 = 9$$
$$\sqrt[3]{t} = 3$$
$$t = 27$$

The height will be 4.1 metres after 27 years.

E.g. 3 A curve has $\frac{d^2y}{dx^2} = 12x$ and when $x = 1$, $\frac{dy}{dx} = 4$ and $y = 7$. Find the equation of the curve.

Working:

$$\frac{dy}{dx} = \int 12x\,dx = 6x^2 + c$$

When $x = 1, \frac{dy}{dx} = 4$: $4 = 6 + c \Rightarrow c = -2$

$$\therefore \frac{dy}{dx} = 6x^2 - 2$$

$y = \int (6x^2 - 2)dx = 2x^3 - 2x + k$, where k is a constant

N.B. Since I used c earlier in the question, a different letter, k, is needed for the 2nd constant.

When $x = 1, y = 7$: $7 = 2 - 2 + k \Rightarrow k = 7$

The equation of the curve is $y = 2x^3 - 2x + 7$.

Video: Finding the constant term

Finding the constant term EQ

Solutions to Starter and E.g.s

Exercise
p299 15C Qu 1i, 2i, 3-10

Summary
Finding the constant term
1. Integrate the function —make sure the "$+c$" is included.
2. Substitute the values given in the question in order to find the value of c.