Finding the Gradient at a Point

Starter

1. **(Review of last lesson)**

Differentiate: (a) $f(x) = -\dfrac{5}{2\sqrt[8]{x^3}}$ (b) $y = \dfrac{15+7x}{8x^3}$

Working: (a) $f(x) = -\dfrac{5}{2\sqrt[8]{x^3}} = -\dfrac{5}{2x^{\frac{3}{8}}} = -\dfrac{5x^{-\frac{3}{8}}}{2}$

$f'(x) = \dfrac{15x^{-\frac{11}{8}}}{16} = \dfrac{15}{16x^{\frac{11}{8}}} = \dfrac{15}{16\sqrt[8]{x^{11}}}$

(b) $y = \dfrac{15+7x}{8x^3} = \dfrac{15}{8x^3} + \dfrac{7x}{8x^3} = \dfrac{15x^{-3}}{8} + \dfrac{7x^{-2}}{8}$

$\dfrac{dy}{dx} = -\dfrac{45x^{-4}}{8} - \dfrac{14x^{-3}}{8} = -\dfrac{45}{8x^4} - \dfrac{7}{4x^3}$

2. Find the gradient of the curve $y = 5x^2 + 4x - 8$ where $x = 2$.

Working: $\dfrac{dy}{dx} = 10x + 4$

When $x = 2$, $\dfrac{dy}{dx} = 10 \times 2 + 4 = 24$

3. Write down a method for finding the gradient of a curve at a particular point.

Working:
1. Differentiate to find the derivative
2. Substitute the $x-$value into the derivative to find the value of the gradient

4. Find the coordinates of the point(s) on $y = (2x-5)(x+1)$ where the gradient is −3.

Working: $y = (2x-5)(x+1) = 2x^2 - 3x - 5$

$\dfrac{dy}{dx} = 4x - 3$

$\dfrac{dy}{dx} = -3$ when $4x - 3 = -3 \quad \Rightarrow \quad x = 0$

When $x = 0$, $y = (0-5)(0+1) = -5$

The gradient is −3 at $(0, -5)$.

5. Write down a method for finding the coordinates point(s) at which the gradient of a curve has a specific value.

Working:
1. Differentiate to find the derivative.
2. Put the derivative equal to the value of the gradient.
3. Solve the equation to find the $x-$coordinate.
4. Substitute the $x-$coordinate into the equation for y (original equation).

E.g. 1 Find the gradient of the curve at the given point:

(a) $s = \sqrt{t}(1 + \sqrt{t})$ when $t = 4$

(b) $y = \dfrac{x^2 - 4}{x}$ when $x = -2$

Working: (a)

$$s = \sqrt{t}(1 + \sqrt{t}) = \sqrt{t} + t = t^{\frac{1}{2}} + t$$

$$\frac{ds}{dt} = \frac{t^{-\frac{1}{2}}}{2} + 1 = \frac{1}{2t^{\frac{1}{2}}} + 1 = \frac{1}{2\sqrt{t}} + 1$$

When $t = 4$, $\dfrac{ds}{dt} = \dfrac{1}{2\sqrt{4}} + 1 = \dfrac{1}{4} + 1 = \dfrac{5}{4}$

(b)

$$y = \frac{x^2 - 4}{x} = \frac{x^2}{x} - \frac{4}{x} = x - 4x^{-1}$$

$$\frac{dy}{dx} = 1 + 4x^{-2} = 1 + \frac{4}{x^2}$$

When $x = -2$, $\dfrac{dy}{dx} = 1 + \dfrac{4}{(-2)^2} = 1 + \dfrac{4}{4} = 2$

E.g. 2 Find the coordinates of the point(s) on the given curve where the gradient has the value specified:

(a) $s = t^3 - 12t + 9$; gradient = 15

(b) $y = 3 - \dfrac{2}{x}$; gradient = $\dfrac{1}{2}$

Working: (a)

$$\frac{ds}{dt} = 3t^2 - 12$$

$\dfrac{ds}{dt} = 15$ when $3t^2 - 12 = 15 \quad \Rightarrow \quad 3t^2 = 27$

$t^2 = 9$ so $\quad \therefore t = \pm 3$

When $t = 3$, $s = 3^3 - 12 \times 3 + 9 = 0$

When $t = -3$, $s = (-3)^3 - 12 \times (-3) + 9 = 18$

The curve has gradient 15 at $(3, 0)$ and $(-3, 18)$

(b)

$$y = 3 - \frac{2}{x} = 3 - 2x^{-1}$$

$$\frac{dy}{dx} = 2x^{-2} = \frac{2}{x^2}$$

$\dfrac{dy}{dx} = \dfrac{1}{2}$ when $\dfrac{2}{x^2} = \dfrac{1}{2} \quad \Rightarrow \quad 4 = x^2 \quad \therefore x = \pm 2$

When $x = 2$, $y = 3 - \dfrac{2}{2} = 2$

When $x = -2$, $y = 3 - \dfrac{2}{(-2)} = 4$

The curve has gradient $\dfrac{1}{2}$ at $(2, 2)$ and $(-2, 4)$

Video: Gradient of a curve at a point

Solutions to Starter and E.g.s

Exercise

p264 13E Qu 3i, 4i, 6i, 11, 14