

## Graph of tangent

### Starter

1. **(Review of last lesson)** Two sides of a triangle have lengths  $(2x + 1)$  and  $(x + 4)$ . Given that the angle between the sides is  $60^\circ$  and that the area of the triangle is  $20\sqrt{3}$ , find the value of  $x$  to 3 s.f..

**Working:** Area of a triangle  $= \frac{1}{2}ab \sin C$ :  $\frac{1}{2}(2x + 1)(x + 4)\sin 60^\circ = 20\sqrt{3}$

Since  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ :  $\frac{1}{2}(2x + 1)(x + 4) \times \frac{\sqrt{3}}{2} = 20\sqrt{3}$

$$(2x + 1)(x + 4) = 80$$

$$2x^2 + 9x - 76 = 0$$

Solving gives  $x = 4.312$  or  $x = -8.812$ .  
Since  $x > 0$ ,  $x = 4.31$  (3 s.f.)

- E.g. 1** (a) Using the tangent graph, find an angle  $\theta$ , where  $0^\circ \leq \theta \leq 360^\circ$ , such that:  
(i)  $\tan 60^\circ = \tan \theta$  (ii)  $\tan 120^\circ = \tan \theta$  (iii)  $\tan 320^\circ = \tan \theta$
- (b) Hence, complete  $\tan x \equiv \tan(\dots)$  with an expression involving  $x$  in the bracket.

**Working:** (a) (i)  $\tan 60^\circ = \tan 240^\circ$   
(ii)  $\tan 120^\circ = \tan 300^\circ$   
(iii)  $\tan 320^\circ = \tan 140^\circ$

(b)  $\tan x \equiv \tan(x \pm 180^\circ)$

- E.g. 2** Using the extended tangent graph of find three values for  $\theta$  angle such that:  
(a)  $\tan 45^\circ = \tan \theta$  (b)  $\tan 150^\circ = \tan \theta$  (c)  $\tan(-160^\circ) = \tan \theta$

**Working:** (a)  $\tan 45^\circ = \tan 225^\circ = \tan 405^\circ = \tan(-135^\circ) = \tan(-315^\circ)$   
Only three angles are required.

(b)  $\tan 150^\circ = \tan(-210^\circ) = \tan(-30^\circ) = \tan 330^\circ$

(c)  $\tan(-160^\circ) = \tan(-340^\circ) = \tan 20^\circ = \tan 200^\circ$

- E.g. 3** (a) Given that  $\tan 40^\circ = 0.839$ , without using a calculator, state the value of:  
(i)  $\tan 220^\circ$  (b)  $\tan(-40^\circ)$  (c)  $\tan 140^\circ$   
(b) Hence state the relationship between  $\tan(-\theta)$  and  $\tan \theta$ .

**Working:** (a) (i)  $\tan 220^\circ = \tan(220^\circ - 180^\circ) = \tan 40^\circ = 0.839$

(ii)  $\tan(-40^\circ) = -\tan 40^\circ = -0.839$

(iii)  $\tan 140^\circ = (\tan 140^\circ - \tan 180^\circ) = \tan(-40^\circ) = -0.839$

(b)  $\tan(-\theta) = -\tan \theta$

**E.g. 4** In a right-angled triangle, the tangent of an angle is given by  $\frac{\text{opp}}{\text{adj}}$ . State the fractions for sine and cosine and by doing  $\frac{\sin \theta}{\cos \theta}$ , find the relationship between sine, cosine and tangent.

**Working:**

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \text{and} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{\frac{\text{opp}}{\text{hyp}}}{\frac{\text{adj}}{\text{hyp}}} = \frac{\text{opp}}{\text{hyp}} \times \frac{\text{hyp}}{\text{adj}} = \frac{\text{opp}}{\text{adj}} = \tan \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

**E.g. 5** By considering the graph of  $y = \tan x$ , decide whether the curve is positive or negative for the range of values in the table.

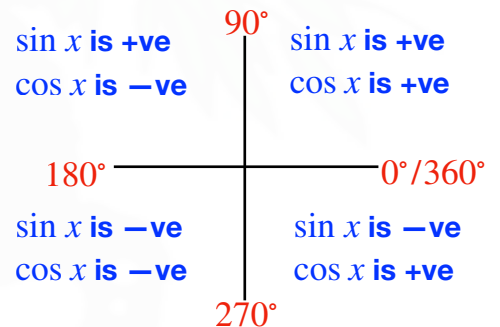
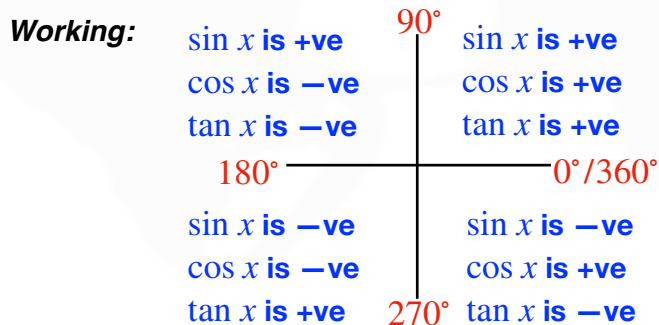
Ratio	$0^\circ < x < 90^\circ$	$90^\circ < x < 180^\circ$	$180^\circ < x < 270^\circ$	$270^\circ < x < 360^\circ$
$\tan x$				

**Working:** The curve is positive when it is above the  $x$ -axis and negative when it is below the  $x$ -axis.

Ratio	$0^\circ < x < 90^\circ$	$90^\circ < x < 180^\circ$	$180^\circ < x < 270^\circ$	$270^\circ < x < 360^\circ$
$\tan x$	Positive	Negative	Positive	Negative

**E.g. 6** Using your table from **E.g. 1**, write in each quadrant whether  $\tan x$  is positive or negative for the range of values within that quadrant.

**N.B.** Negative angles are measured clockwise.



**Video:** [Trigonometric graphs](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

p176 10B Qu 1b, 2i, 3, 4, 5-6, (7 red)