Hypothesis testing with the Binomial distribution

Starter

1. ***(Review of last lesson)*** Given that $X \sim \text{B}\left(60, \dfrac{1}{6}\right)$, use the BinomialCD function on your calculator to find:

(a)	$P(X = 0)$	(b)	$P(X \leq 1)$	(c)	$P(X \leq 2)$	(d)	$P(X \leq 3)$
(e)	$P(X \leq 4)$	(f)	$P(X \leq 5)$	(g)	$P(X \geq 15)$	(h)	$P(X \geq 16)$

Working:

(a) $P(X = 0) = 0.0000177$ (b) $P(X \leq 1) = 0.000231$

(c) $P(X \leq 2) = 0.00149$ (d) $P(X \leq 3) = 0.00635$

(e) $P(X \leq 4) = 0.0202$ (f) $P(X \leq 5) = 0.0512$

(g) $P(X \geq 15) = 1 - P(X \leq 14) = 1 - 0.935 = 0.065$

(h) $P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.966 = 0.034$

E.g. 1 At the start of last season, Erik carried out a football team supporter's survey and his results suggested that $30\,\%$ supported Team A. After a poor season Erik now believes their support will have gone down and carries out another survey. From a sample of 40 people, six people said they support Team A. Is Erik right (use a $4\,\%$ significance level)?

Working:

Last season, $P(\text{support Team A}) = 0.3$.

If this is still true with the sample of 40 people, then $X \sim \text{B}\,(40, 0.3)$.

Erik now believes $P(\text{support Team A}) < 0.3$

Consider values that are ***at least as extreme as the observed value.***

$P(X \leq 6) = 0.0238 < 0.04$

The result is significant.

There is ***sufficient evidence*** to suggest that support for Team A has gone down.

E.g. 2 Results from previous years indicate that $91\,\%$ of students pass the Grade 8 practical music exam. Following coronavirus, the music organisation that runs the exam wants to know whether this has changed. Recent results indicate that out of 318 students who took the Grade 8 practical music exam, 300 passed. Is there evidence, at the $5\,\%$ level, that the pass rate has changed? Try to give an explanation for your conclusion.

Working:

Previously $P(\text{pass}) = 0.91$

If it is still true with the sample of 318 students, then $X \sim \text{B}\,(318, 0.91)$.

Consider values that are ***at least as extreme as the observed value.***

Since $\dfrac{300}{318} > 0.91$, calculate $P(X \geq 300)$:

$P(X \geq 300) = 1 - P(X \leq 299) = 1 - 0.981 = 0.019$

Since only a change is considered, ***halve the significance level*** when comparing it with the calculated probability.

$P(X \geq 300) = 0.019 < 0.025$.

So the result is significant. There is ***sufficient evidence*** to suggest that the pass rate for the Grade 8 practical music exam ***has changed***.

This may be because students had more time to practice their musical instrument during lockdown.

E.g. 3 A manufacturer claims to have a 90% germination rate for their seeds. A gardener believes it is less and to test the theory plants 100 seeds from which 84 germinate. Test the manufacturer's claim at the 5% level.

Working: "believes it is less" $\Rightarrow$ one-tailed test

$H_0 : p = 0.9$ where p is the proportion of seeds that germinate.

$H_1 : p < 0.9$

Under H_0, $X \sim \text{B}(100, 0.9)$.

$\alpha = 0.05$

$P(X \leq 84) = 0.0399 < 0.05 = \alpha$

The result is significant so reject H_0.

There is sufficient evidence to suggest, at the 5% level, that the proportion of seeds that germinate as stated by the manufacturer is too high.

E.g. 4 A political party's support is stagnant at 3 out of 10 people. After their party conference, they believe their support has increased. Would you accept this assertion if a further survey revealed that 36 people in a random sample of 95 people supported the party? Test at the 3% level.

Working: "believe their support has increased" $\Rightarrow$ one-tailed test

$H_0 : p = 0.3$ where p is the proportion of support for the political party

$H_1 : p > 0.3$

Under H_0, $X \sim \text{B}(95, 0.3)$.

$\alpha = 0.03$

$P(X \geq 36) = 1 - P(X \leq 35) = 1 - 0.939 = 0.061 \nless 0.03$

The result is not significant so do not reject H_0.

There is insufficient evidence to suggest, at the 3% level, that the proportion of support for the political party has increased.

E.g. 5 A driving instructor changes their car. With the previous car, 73% of their students passed the practical side of the driving test at the first attempt. After a period of time with the new car, 16 out of 18 passed first time. Determine, at the 10% significance level whether the new car has made a difference in the pass rate.

Working: "made a difference in the pass rate" $\Rightarrow$ two-tailed test

$H_0 : p = 0.73$ where p is the proportion of students who pass the driving test first time

$H_1 : p \neq 0.73$

Under H_0, $X \sim \text{B}(18, 0.73)$.

Although $\alpha = 0.1$, it is a two-tailed test so we use 0.05.

Since $\frac{16}{18} > 0.73$ find $P(X \geq 16)$.

$P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.901 = 0.099 \nless 0.05$

The result is not significant so do not reject H_0.

There is insufficient evidence to suggest, at the 10% level, that the pass rate for the practical side of the driving test has changed.

E.g. 6 A lottery company claims that 15% of all tickets sold win a prize. Believing the lottery to be fraudulent, the Office for Trading Standards buys 250 tickets and wins 27 prizes. Carry out a test at the (a) 2% and (b) 5% to decide whether this indicates the lottery is fraudulent.

Working: "believing the lottery to be fraudulent" $\Rightarrow$ one-tailed test

$H_0 : p = 0.15$ where p is the proportion of winning tickets.

$H_1 : p < 0.15$

Under H_0, $X \sim$ B $(250, 0.15)$.

$\alpha = 0.02$

(a) $P(X \leq 27) = 0.034 \nless 0.02 = \alpha$

The result is not significant so do not reject H_0.

There is insufficient evidence to suggest, at the 2% level, that the lottery is fraudulent.

(b) $P(X \leq 27) = 0.034 < 0.05 = \alpha$

The result is significant so reject H_0.

There is sufficient evidence to suggest, at the 5% level, that the lottery is fraudulent.

Different conclusions can be reached ***depending on the significance level*** of the hypothesis test. In addition, **another sample** may also lead to a ***different outcome***.

Video: **Hypothesis test with the Binomial distribution**

Video: **Lower tail test**

Video: **Upper tail test**

Video: **Two-tailed tests**

Solutions to Starter and E.g.s

Exercise

p400 18B Qu 1i, 2i, 3-11, (12-13 red)