Interpreting Derivatives and Second Derivatives

Starter

1. **(Review of last lesson)** Find the exact value of the gradient of the tangent at the point where $x = 2$ on the curve $y = (2 - \sqrt{x})^2$.

Working:

$$y = (2 - \sqrt{x})^2 = 4 - 4\sqrt{x} + x = = 4 - 4x^{\frac{1}{2}} + x$$

$$\frac{dy}{dx} = -2x^{-\frac{1}{2}} + 1 = 1 - \frac{2}{x^{\frac{1}{2}}} = 1 - \frac{2}{\sqrt{x}}$$

When $x = 2$, $\frac{dy}{dx} = 1 - \frac{2}{\sqrt{2}} = 1 - \sqrt{2}$

2. Find the second derivative of the following functions:

(a) $y = x^3$ (b) $y = x(4x^2 - x)$ (c) $y = 5x - 2$

Working:

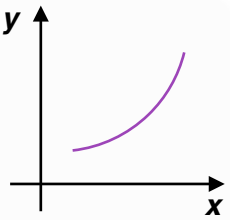
(a) $\frac{dy}{dx} = 3x^2 \quad \Rightarrow \quad \frac{d^2y}{dx^2} = 6x$

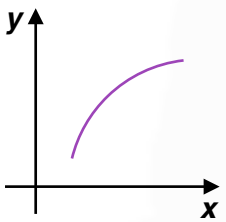
(b) $y = x(4x^2 - x) = 4x^3 - x^2$

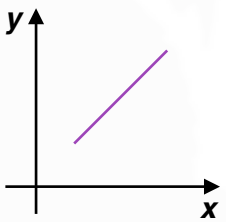
$\frac{dy}{dx} = 12x^2 - 2x \quad \Rightarrow \quad \frac{d^2y}{dx^2} = 24x - 2$

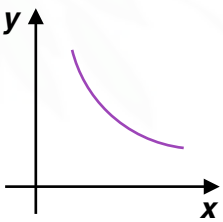
(c) $\frac{dy}{dx} = 5 \quad \Rightarrow \quad \frac{d^2y}{dx^2} = 0$

E.g. 1 Write down the signs of $f'(x)$ and $f''(x)$ for the following graphs of $y = f(x)$.

(a) 

(b) 

(c) 

(d) 

Working:

(a) Curve is going up so $f'(x) > 0$
The gradient of the curve is increasing so $f''(x) > 0$

(b) Curve is going up so $f'(x) > 0$
The gradient of the curve is decreasing so $f''(x) < 0$

(c) Line is going up so $f'(x) > 0$
As seen from 2(c) of the starter, the second derivative of straight line graphs is zero i.e. $f''(x) = 0$

(d) Curve is going down so $f'(x) < 0$
The gradient of the curve is increasing so $f''(x) > 0$

E.g. 2 Find the value of the 2nd derivative at the given value of x:

(a) $f(x) = x^3 - x^2,\ x = 3$ (b) $y = x\sqrt{x} - \frac{1}{x},\ x = 4$

Working: (a) $f'(x) = 3x^2 - 2x \quad \Rightarrow \quad f''(x) = 6x - 2$
$f''(3) = 6 \times 3 - 2 = 16$

(b) $y = x\sqrt{x} - \frac{1}{x} = x^{\frac{3}{2}} - x^{-1}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + x^{-2} \quad \Rightarrow$

$\frac{d^2y}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}} - 2x^{-3} = \frac{3}{4x^{\frac{1}{2}}} - \frac{2}{x^3} = \frac{3}{4\sqrt{x}} - \frac{2}{x^3}$

When $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{4\sqrt{4}} - \frac{2}{4^3} = \frac{3}{8} - \frac{1}{32} = \frac{11}{32}$

Video: Second derivative

Solutions to Starter and E.g.s

Exercise
p264 13E Qu 1, 2i, 5i