Kinematics and Calculus

Starter

1. **(Review of AS material)**

Find the gradient of the curve $x = t + 3t^2$ at the point where $t = -2$.

Working: $x = t + 3t^2 \Rightarrow \dfrac{dx}{dt} = 1 + 6t$

When $t = -2$: $\dfrac{dx}{dt} = 1 + 6 \times (-2) = -11$

2. **(Review of AS material)**

Given that $\dfrac{dv}{dt} = t^3 + 2t - 1$ and that $v = 13$ when $t = 2$, find v as a function of t.

Working: $v = \int (t^3 + 2t - 1)dt = \dfrac{1}{4}t^4 + t^2 - t + c$

When $t = 2, v = 13$: $13 = \dfrac{1}{4} \times 2^4 + 2^2 - 2 + c \Rightarrow c = 7$

$\therefore v = \dfrac{1}{4}t^4 + t^2 - t + 7$

E.g. 1 An object's displacement in metres at t second is given by $x = 2t^3 - 4t^2 + 3$.

(a) Calculate the object's displacement at time $t = 3$.
(b) Find the object's velocity equation.
(c) What is its velocity at time $t = 3$?
(d) Find its initial acceleration.

Working: (a) When $t = 3$: $x = 2 \times 3^3 - 4 \times 3^2 + 3 = 21$ m

(b) $v = \dfrac{dx}{dt} = 6t^2 - 8t$

(c) When $t = 3$: $v = \dfrac{dx}{dt} = 6 \times 3^2 - 8 \times 3 = 30$ m/s

(d) $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = 12t - 8$

When $t = 0$: $a = -8$ m/s².

E.g. 2 During braking the speed of a car is modelled by $v = 40 - 2t^2$ (in m/s) until it stops moving.

(a) How long is it before the car stops moving?

(b) How far does it move before it stops? Give your answer to the nearest metre.

Working:

(a) The car stops moving when $v = 0$:

$$40 - 2t^2 = 0$$
$$t^2 = 20$$
$$t = 2\sqrt{5}$$

It takes $2\sqrt{5} \approx 4.47$s to stop moving

(b) Distance travelled $= \int_0^{2\sqrt{5}} (40 - 2t^2)dt$

$$= \left[40t - \frac{2}{3}t^3\right]_0^{2\sqrt{5}}$$
$$= \left(40 \times 2\sqrt{5} - \frac{2}{3} \times (2\sqrt{5})^3\right) - (0 - 0)$$
$$= 80\sqrt{5} - \frac{80}{3}\sqrt{5}$$
$$\approx 119.26 \text{ m}$$

The car stops after 119 m.

Video: Linear motion with variable acceleration

Linear motion with variable acceleration EQ

Solutions to Starter and E.g.s

Exercise

p431 19C Qu 1i, 3i, 5i, 6-9