

Laws of Indices

Starter

1. (Review of GCSE material)

Simplify:

(a)	$3a^{-3} \times 2a^7$	(b)	$\frac{4c^2}{8(c^5)^2}$
(c)	$(4d^2)^3$	(d)	$(3x)^4 \times (2x)^4$

Working: (a) $3a^{-3} \times 2a^7 = (3 \times 2)a^{-3+7} = 6a^4$

(b) $\frac{4c^2}{8(c^5)^2} = \frac{4c^2}{8c^{5 \times 2}} = \frac{4c^2}{8c^{10}} = \frac{1}{2c^8}$

(c) $(4d^2)^3 = 4^3 d^{2 \times 3} = 64d^6$

(d) $(3x)^4 \times (2x)^4 = (6x^2)^4 = 6^4 x^{2 \times 4} = 1296x^8$

2. (Review of GCSE material)

Without using a calculator, evaluate: (a) $16^{\frac{1}{4}}$ (b) $64^{-\frac{2}{3}}$ (c) $7a^0$

Working: (a) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

(b) $64^{-\frac{2}{3}} = \frac{1}{64^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{64})^2} = \frac{1}{4^2} = \frac{1}{16}$

(c) $7a^0 = 7 \times 1 = 7$ *since any number to the power of zero is 1*

3. (Review of GCSE material)

Correct the mistakes in this simplification: $(3x^2)^3 \times 2x^{-4} = 3x^6 \times 2x^{-4} = 5x^{10}$

Working: $(3x^2)^3 \times 2x^{-4} = 27x^6 \times 2x^{-4} = 54x^2$

E.g. 1 Without a calculator, find the value of $\left(\frac{2}{5}\right)^{-3}$.

Working: $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{5^3}{2^3} = \frac{125}{8}$

E.g. 2 Without a calculator, find the value of $8^{\frac{2}{3}}$.

Working: $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

E.g. 3 Express $2^5 \times 3^5$ in the form x^y .

Working: $2^5 \times 3^5 = (2 \times 3)^5 = 6^5$

E.g. 4 Without a calculator, evaluate:

	(a) $27^{\frac{4}{3}}$	(b) $\left(3\frac{3}{8}\right)^{\frac{2}{3}}$
	(c) $9^{-\frac{3}{2}}$	(d) $\left(2\frac{1}{4}\right)^{-\frac{1}{2}}$

Working: (a) $27^{\frac{4}{3}} = (27^{\frac{1}{3}})^4 = (\sqrt[3]{27})^4 = 3^4 = 81$

(b) $\left(3\frac{3}{8}\right)^{\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\frac{\sqrt[3]{27}}{\sqrt[3]{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

(c) $9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{\sqrt{9^3}} = \frac{1}{3^3} = \frac{1}{27}$

(d) $\left(2\frac{1}{4}\right)^{-\frac{1}{2}} = \left(\frac{9}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{2}{3}$

E.g. 5 Simplify:

(a) $9a^{-2} \times \frac{1}{3}a^{-3} \div \frac{2}{3}a^{-2}$	(b) $\frac{6x^4 + 10x^6}{2x}$
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Working: (a) $9a^{-2} \times \frac{1}{3}a^{-3} \div \frac{2}{3}a^{-2} = 9a^{-2} \times \frac{1}{3}a^{-3} \times \frac{3}{2}a^2$
 $= 9 \times \frac{1}{3} \times \frac{3}{2} \times a^{-2+-3+2} = \frac{9}{2a^3}$

(b) $\frac{6x^4 + 10x^6}{2x} = \frac{2x^4(3 + 5x^2)}{2x} = x^3(3 + 5x^2)$

E.g. 6 Solve the equation $4^x \times 3^{2x} = 6$.

Working: Since $4 = 2^2$: $2^{2x} \times 3^{2x} = 6$
 As the powers are the same, the bases can be multiplied: $6^{2x} = 6^1$
 Equating powers: $2x = 1$
 $x = \frac{1}{2}$

E.g. 7 Given that $2^{x+y} = 1$ and $10^{3x-y} = 100$, find x and y . Hence find 5^{y-x} and x^y .

Video: [Classwiz - solving simultaneous equations](#)

Working:

$$\begin{aligned} 2^{x+y} = 1 &: & x + y &= 1 \\ 10^{3x-y} = 100 & \Rightarrow & 10^{3x-y} &= 10^2 & \therefore 3x - y &= 2 \\ \text{Solving simultaneously:} & & x &= 0.5, y &= -0.5 \\ 5^{y-x} &= 5^{-1} &= \frac{1}{5} \\ x^y &= \left(\frac{1}{2}\right)^{-\frac{1}{2}} &= 2^{\frac{1}{2}} &= \sqrt{2} \end{aligned}$$

Video: [Simplifying using the laws of indices](#)

Video: [Equations involving indices](#)

[Solutions to Starter and E.g.s](#)

Exercise

p20 2A Qu 1ace..., 2ace..., 3ac, 4, 5ace, 6(i), 7-13 odd, 14-16 (Not 17 - need logs)

[Core 1&2 p110 7B, p117 7C]