Logarithms

Starter

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1. (Review of last lesson) Solve $3^{2x+1} - 12 \times 3^{x+2} + 729 = 0$.
   Hint: prepare the equation so that we can "Let $u = 3^x$"

Working:

$$3 \times 3^{2x} - 12 \times 3^2 \times 3^x + 729 = 0$$
$$3 \times 3^{2x} - 108 \times 3^x + 729 = 0$$
Let $u = 3^x$: $3u^2 - 108u + 729 = 0$
$$u^2 - 36u + 243 = 0$$
$u = 9$ or $u = 27$
$3^x = 9$ or $3^x = 27$
$x = 2$ or $x = 3$

2. Solve these equations: (a) $2^{x+4} = \dfrac{16^{3x}}{8^{1-5x}}$ (b) $2^x = 9$.
   N.B. For (a), express 16 and 8 as powers of 2, then use the laws of indices.
   If you can't do (b), don't spend *too* much time on it.

Working: (a) $16 = 2^4$ and $8 = 2^3$

$$2^{x+4} = \frac{\left(2^4\right)^{3x}}{\left(2^3\right)^{1-5x}} \quad \Rightarrow \quad 2^{x+4} = \frac{2^{12x}}{2^{3-15x}}$$
$$2^{x+4} = 2^{27x-3}$$
Equating powers of 2: $x + 4 = 27x - 3$
$$7 = 26x$$
$$x = \frac{7}{26}$$

(b) To solve (b) we need ***logarithms***

E.g. 1 By using the phrase above, state the values of:
(a) $\log_a a$ (b) $\log_a 1$ (c) $\log_2 8$ (d) $\log_3 9$ (e) $\log_{25} 5$.
Give reasons for your answers using indices.

Working:

(a) $\log_a a$ $\Rightarrow$ To what power must a be raised to get a
$\log_a a = 1$ since $a^1 = a$

(b) $\log_a 1$ $\Rightarrow$ To what power must a be raised to get 1
$\log_a 1 = 0$ since $a^0 = 1$

(c) $\log_2 8$ $\Rightarrow$ To what power must 2 be raised to get 8
$\log_2 8 = 3$ since $2^3 = 8$

(d) $\log_3 9$ $\Rightarrow$ To what power must 3 be raised to get 9
$\log_3 9 = 2$ since $3^2 = 9$

(e) $\log_{25} 5$ $\Rightarrow$ To what power must 5 be raised to get 25
$\log_{25} 5 = \dfrac{1}{2}$ since $25^{\frac{1}{2}} = 5$

E.g. 2 Without using a calculator, state the values of:

(a) $\log_2 16$ (b) $\log 10000$ (c) $\log_5 \frac{1}{5}$ (d) $\log_7 \sqrt{7}$

Remember: ***if no base is written, assume it is base 10***

Working:

(a) $\log_2 16 \Rightarrow$ To what power must 2 be raised to get 16

$\log_2 16 = 4$

(b) If no base is written, it is base 10

$\log 10000 \Rightarrow$ To what power must 10 be raised to get 10000

$\log 10000 = 4$

(c) $\log_5 \frac{1}{5} \Rightarrow$ To what power must 5 be raised to get $\frac{1}{5}$

$\log_5 \frac{1}{5} = -1$

(d) $\log_7 \sqrt{7} \Rightarrow$ To what power must 7 be raised to get $\sqrt{7}$

$\log_7 \sqrt{7} = \frac{1}{2}$

E.g. 3 Rewrite in logarithm form:

(a) $4^3 = 64$ (b) $9^{\frac{1}{2}} = 3$ (c) $\sqrt[3]{8} = 2$ (d) $10^{-4} = 0.0001$

Working:

(a) $4^3 = 64 \Leftrightarrow \log_4 64 = 3$

(b) $9^{\frac{1}{2}} = 3 \Leftrightarrow \log_9 3 = \frac{1}{2}$

(c) $\sqrt[3]{8} = 2 \Leftrightarrow \log_8 2 = \frac{1}{3}$

(d) $10^{-4} = 0.0001 \Leftrightarrow \log 0.0001 = -4$

N.B. With base 10, there is no need to write the base

E.g. 4 Rewrite in index form:

(a) $\log_3 81 = 4$ (b) $\log_{125} 5 = \frac{1}{3}$ (c) $\log_2 \frac{1}{32} = -5$ (d) $\ln a = 3b$

Working:

(a) $\log_3 81 = 4 \Leftrightarrow 3^4 = 81$

(b) $\log_{125} 5 = \frac{1}{3} \Leftrightarrow 125^{\frac{1}{3}} = 5$

(c) $\log_2 \frac{1}{32} = -5 \Leftrightarrow 2^{-5} = \frac{1}{32}$

(d) $\ln a = 3b \Leftrightarrow e^{3b} = a$

N.B. Remember ln means $\log_e$

E.g. 5 By writing the following in index notation, find the value of x:

(a) $\log_x 49 = 2$ (b) $\log_4 x = 3$ (c) $\log_x 7 = \frac{1}{3}$

Working: (a) $x^2 = 49$ $x = 7$

(b) $4^3 = x$ $x = 64$

(c) $x^{\frac{1}{3}} = 7$ $x = 343$

Video: What do we mean by a logarithm?
Video: Natural logarithms

Solutions to Starter and E.g.s

Exercise
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