Multi-Stage Problems

Starter

1. **(Review of last lesson)** A youth is playing with a ball in a garden surrounded by a wall 2.5 m high and kicks the ball vertically up from a height of 0.4 m with a speed of 14 m/s. For how long is the ball above the height of the wall? Give your answer to 2 s.f.

Working: ***Ball is going up and down so the positive direction is upwards and*** $a = -9.8$.

$u = 14, a = -9.8, s = 2.5 - 0.4 = 2.1, t = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $\quad 14t + \frac{1}{2} \times (-9.8) \times t^2 > 2.1$

$4.9t^2 - 14t - 2.1 < 0$

Solving $4.9t^2 - 14t - 2.1 = 0$ gives $t \approx 0.159$ and $t \approx 2.698$

$t \approx 0.159$ $\Rightarrow$ ball passes top of wall going up

$t \approx 2.698$ $\Rightarrow$ ball passes wall coming down

Time above the wall is $2.698 - 0.159 = 2.5$ s (2 s.f.)

2. **(Review of last lesson)** A competitor is attempting a dive that is 6 m above the water. She leaves the springboard with an upward velocity of 7 m/s. Taking $g = 10$, find the speed at which the diver enters the water and the time for which she is in the air.

Working: ***The diver is going up and down so the positive direction is upwards and*** $a = -10$.

The water is below where she set off so $s = -6$

$u = 7, a = -10, s = -6, v = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $\quad v^2 = 7^2 + 2 \times (-10) \times (-6)$

$v^2 = 169$

$v = 13$

N.B. Avoid using a previous answer if possible.

$u = 7, a = -10, s = -6, t = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $\quad -6 = 7t + \frac{1}{2} \times (-10) \times t^2$

$5t^2 - 7t - 6 = 0$

$(5t + 3)(t - 2) = 0$

$\therefore t = -\frac{3}{5}$ or $t = 2$

Since $t \geq 0, t = 2$

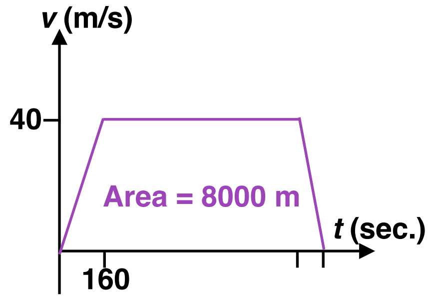
The diver hits the water with speed 13 m/s after 2 s.

E.g. 1 A train has a maximum speed of 144 **km/h** which it can achieve at an acceleration of 0.25 m/s². With its brakes applied fully, the train as a deceleration of 0.5 m/s². Two stations are 8 km apart. The trains stop at both stations.

(a) What is the shortest time for the train to travel between these two stations?

(b) How is your answer to (a) changed if there is a restriction on speed, between the two stations, of 72 km/h?

Working: (a) 144 km/h ≡ 40 m/s



Accelerating:

$a = 0.25,\ u = 0,\ v = 40,\ t = ?$

No s ⇒ $v = u + at$

$40 = 0 + 0.25t$

$t = 160$

$a = 0.25,\ u = 0,\ v = 40,\ s = ?$

No t ⇒ $v^2 = u^2 + 2as$

$40^2 = 0^2 + 2 \times 0.25 \times s$

$s = 3200$ ***distance travelled while accelerating***

Decelerating:

$a = -0.5,\ u = 40,\ v = 0,\ t = ?$

No s ⇒ $v = u + at$

$0 = 40 + (-0.5) \times t$

$t = 80$

$a = -0.5,\ u = 40,\ v = 0,\ s = ?$

No t ⇒ $v^2 = u^2 + 2as$

$0^2 = 40^2 + 2 \times (-0.5) \times s$

$s = 1600$ ***distance travelled while decelerating***

So distance travelled at constant speed = 8000 − 3200 − 1600 = 3200

Time taken to travel 3200 m at 40 m/s is 80 s.

Total time between stations 160 + 80 + 80 = 300 s

i.e. 5 minutes 20 seconds

(b) 72 km/h ≡ 20 m/s

Accelerating:

The time to 20 m/s will be 80 s ***half the previous time***

$a = 0.25,\ u = 0,\ v = 20,\ s = ?$

No t ⇒ $v^2 = u^2 + 2as$

$20^2 = 0^2 + 2 \times 0.25 \times s$

$s = 800$ ***distance travelled while accelerating***

Decelerating:

The time to stop will be 40 s ***half the previous time***

$a = -0.5,\ u = 20,\ v = 0,\ s = ?$

No t ⇒ $v^2 = u^2 + 2as$

$0^2 = 20^2 + 2 \times (-0.5) \times s$

$s = 400$ ***distance travelled while decelerating***

So distance travelled at constant speed = 8000 − 800 − 400 = 6800

Time taken to travel 6800 m at 20 m/s is 340 s.

Total time between stations 80 + 40 + 340 = 460 s

i.e. 7 minutes 40 seconds

the increase in time is 7m 40s — 5m 20s = 2 minutes 20 seconds

Video: Multi-stage problem

Solutions to Starter and E.g.s

Exercise
p475 20D Qu 1-8