Proof by deduction

Starter

1. *(Review of last lesson)* Prove that product of two non-equal irrational numbers is not always irrational.

Working: $\sqrt{2}$ and $3\sqrt{2}$ are both irrational

$\sqrt{2} \times 3\sqrt{2} = 6$ which is not irrational since $6 = \frac{6}{1}$.

Therefore, the product of two non-equal irrational numbers is not always irrational

2. (a) Give an algebraic expression for the general form of an even number involving x.
(b) Give an algebraic expression for the general form of an odd number involving y.
(c) Hence prove that the product of an even and an odd number is always even.

Working:

(a) $2x$

(b) $2y + 1$

(c) $2x(2y + 1) = 4xy + 2x = 2(2xy + x) = 2k$ where k is the integer $2xy + x$. Since $2k$ is the form of an even number, the product of an even and an odd number is always even.

E.g. 1 Prove that the mean of five consecutive numbers is two more than the smallest number.

Working: Let the five consecutive numbers be x, $x + 1$, $x + 2$, $x + 3$ and $x + 4$.

$$\frac{x + x + 1 + x + 2 + x + 3 + x + 4}{5} = \frac{5x + 10}{5} = x + 2$$

Since $x + 2$ is two more than the smallest number, the mean of five consecutive numbers is two more than the smallest number.

E.g. 2 Prove that for any integer x, $(x + 7)^2 - x(x - 1) - 1$ is divisible by 3.

Working:

$$\begin{aligned}(x + 7)^2 - x(x - 1) - 1 &= x^2 + 14x + 49 - x^2 + x - 1 \\ &= 15x - 48 \\ &= 3(5x - 16)\end{aligned}$$

which is divisible by 3.

E.g. 3 Mike claims that $n^2 + 4n + 3 > 0$ for all values of n.
(a) Disprove this statement by counter-example.
(b) Use proof by deduction to show that Mike is wrong.

Working:

(a) When $n = -2$, $n^2 + 4n + 3 = (-2)^2 + 4 \times (-2) + 3 = -1 \not> 0$

(n) $n^2 + 4n + 3 = (n + 2)^2 - 4 + 3 = (n + 2)^2 - 1 \not> 0$ when $n = -2$

E.g. 4 A number that is divisible by 5 could be written $5n$.

(a) Write down the algebraic form of a number that leaves a remainder 2 when divided by 5.

(b) Prove that if a number leaves a remainder 2 when divided by 5, its square leaves a remainder 4.

Working: (a) $5n + 2$

(b) $(5n + 2)^2 = 25n^2 + 20n + 4 = 5(5n^2 + 4n) + 4 = 5k + 4$
which is the form of a number that would leave a remainder of 4 when divided by 5.

Video: **Proof by exhaustion and deduction**

Solutions to Starter and E.g.s

Exercise
p11 1D Qu 1-9, (10-12 red)