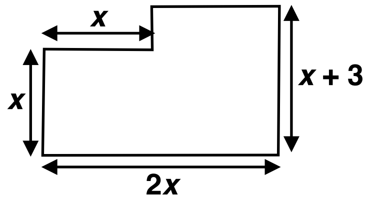
Quadratic Graphs

Starter

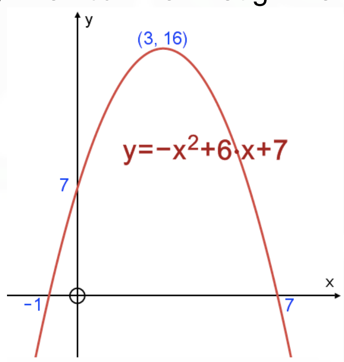
1. **(Review of last lesson)** The shape has an area of 44 units². Find the value of x.



Working: $x^2 + x(x+3) = 44$
$2x^2 + 3x - 44 = 0$
$(2x+11)(x-4) = 0$
$x = -5.5$ or $x = 4$
Since $x > 0$, $x = 4$

E.g. 1 Sketch the graph of $y = -x^2 + 6x + 7$, indicating the coordinates of axes intercepts and the turning point. Give the equation of the line of symmetry.

Working: **Roots**: solve $7 + 6x - x^2 = 0$: $(1+x)(7-x) = 0$
$\therefore$ roots at $x = -1$ and $x = 7$
Concave-up or down: the coefficient of x^2 is $-1 < 0$, concave-down
$y-$**intercept**: when $x = 0$ so $y = 7$
Turning point (half way between the roots): i.e. $x = \frac{-1+7}{2} = 3$
When $x = 3$, $y = -3^2 + 6 \times 3 + 7 = 16$ so TP at $(3, 16)$
Line of symmetry: vertical line through the turning point $x = 3$

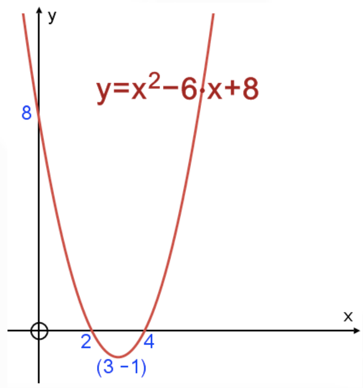


E.g. 2 Sketch the graphs of: (a) $y = x^2 - 6x + 8$ (b) $y = 2(3-x)(1+x)$.
Indicating the coordinates of axes intercepts and the turning point. Give the equation of the line of symmetry.

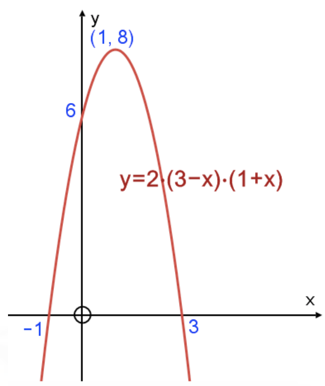
Working: (a) **Roots**: solve $x^2 - 6x + 8 = 0$: $(x-2)(x-4) = 0$
$\therefore$ roots at $x = 2$ and $x = 4$
Concave-up or down: the coefficient of x^2 is $1 > 0$, concave-up
$y-$**intercept**: when $x = 0$ so $y = 8$
Turning point (half way between the roots): i.e. $x = \frac{2+4}{2} = 3$
When $x = 3$, $y = 3^2 - 6 \times 3 + 8 = -1$ so TP at $(3, -1)$
Line of symmetry: vertical line through the turning point $x = 3$

(b) **Roots**: solve $2(3-x)(1+x)=0$:
$(3-x)$ is a factor so $x=3$ is a root
$(1+x)$ is a factor so $x=-1$ is a root
$\therefore$ roots at $x=-1$ and $x=3$
Concave-up or down: by expanding the brackets, it can be seen that the coefficient of x^2 is $-1<0$, concave-down
$y-$**intercept**: when $x=0$ so $y=2\times3\times1=6$
Turning point (half way between the roots): i.e. $x=\dfrac{-1+3}{2}=1$
When $x=1$, $y=2(3-1)(1+2)=8$ so TP at $(1,8)$
Line of symmetry: vertical line through the turning point $x=3$

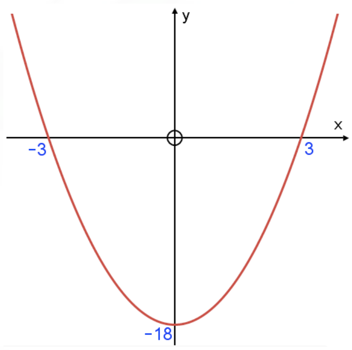
(a)



(b)



E.g. 3 The sketch is a quadratic function of the form $y=ax^2+bx+x$. Find the value of a, b and c

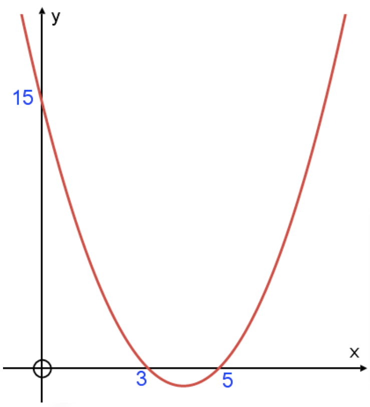


Working:

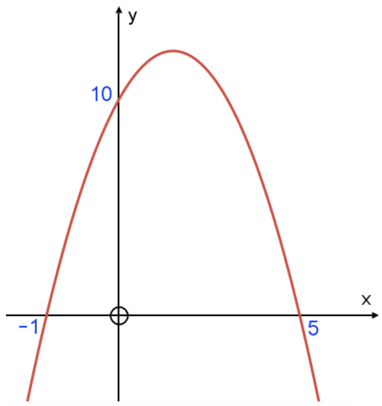
Root at $x=-3$ so $(x+3)$ is a factor
Root at $x=3$ so $(x-3)$ is a factor
Equation is $y=k(x-3)(x+3)$
Curve passes through (0, −18): $-18=k\times-3\times3 \quad \therefore k=2$
Equation is $y=2(x-3)(x+3)$
Expanding gives $y=2x^2-18$
So $a=2$, $b=0$ and $c=-18$

E.g. 4 These sketches are graphs of quadratic functions of the form $y = ax^2 + bx + x$. Find the values of a, b and c for each function.

(a)



(b)



Working: (a) Roots at $x = 3$ and $x = 5$ so factors are $(x - 3)$ and $(x - 5)$
$y = k(x - 3)(x - 5)$
When $x = 0$, $y = 15$ so $15 = k(0 - 3)(0 - 5)$ i.e. $k = 1$
Expand the brackets: $y = x^2 - 8x + 15$
So $a = 1, b = -8, c = 15$

(b) Roots at $x = -1$ and $x = 5$ so factors are $(x + 1)$ and $(x - 5)$
$y = k(x + 1)(x - 5)$
When $x = 0$, $y = 10$ so $10 = k(0 + 1)(0 - 5)$ i.e. $k = -2$
$\therefore y = -2(x + 1)(x - 5)$
Expanding: $y = -2(x^2 - 4x - 5) = -2x^2 + 8x + 10$
$a = -2, b = 8, c = 10$

E.g. 5 The graph of $y = ax^2 + bc + c$ has a minimum at (5, −3) and passes through (4, 0). Find the values of a, b and c.

Working: The minimum, at $x = 5$ is half-way between the roots
If one root is at $x = 4$, the other root must be at $x = 6$
So the curve is of the form $y = k(x - 4)(x - 6)$
Substitute (5, −3): $-3 = k(5 - 4)(5 - 6) \therefore k = 3$
Expanding $y = 3(x - 4)(x - 6)$ gives $a = 3, b = -30, c = 72$

Video: Sketching quadratics

Solutions to Starter and E.g.s

Exercise
p33 3B Qu 1, 2i, 3