

Quadratic Inequalities

Starter

1. **(Review of GCSE material)** Solve the inequalities. Express your answers in set notation.

(a) $\frac{43 - 3x}{7} < 4$

(b) $7 \leq 5x - 8 \leq 22$

Working: (a) $43 - 3x < 28$

Either: $-3x < -15$ or $43 < 3x + 28$
 $x > 5$ $15 < 3x$
 $5 < x$

In set notation: $\{x : x > 5\}$

(b) $15 \leq 5x \leq 30$

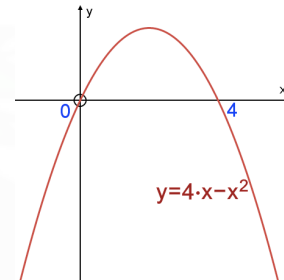
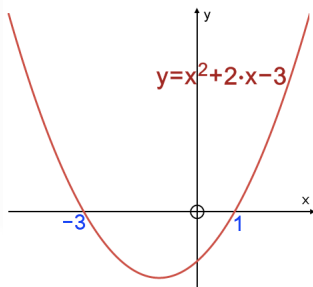
$3 \leq x \leq 6$

In set notation: $\{x : x \geq 3\} \cap \{x : x \leq 6\}$

2. Use the graphs given to solve the quadratic inequalities, expressing your answers in set notation:

(a) $x^2 + 2x - 3 < 0$

(b) $4x - x^2 < 0$



Working: (a) $< 0 \Rightarrow$ where the curve is below the x -axis
 $-3 < x < 1$
 $\{x : x > -3\} \cap \{x : x < 1\}$

(b) $< 0 \Rightarrow$ where the curve is below the x -axis
 $x < 0$ and $x > 4$
 $\{x : x < 0\} \cup \{x : x > 4\}$

E.g. 1 Solve the inequality $x^2 + x > 2$.

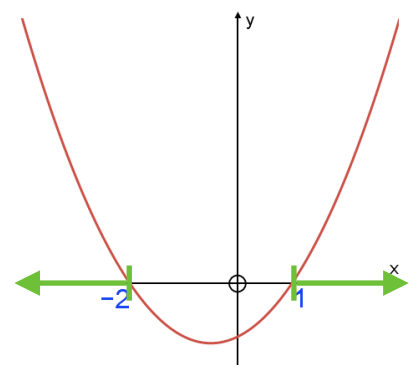
Working: Rearrange $x^2 + x - 2 > 0$
 Solve $x^2 + x - 2 = 0$ to find the roots:
 $(x + 2)(x - 1) = 0$

Roots are $x = -1$ and $x = 2$

Coefficient of x^2 is +ve so concave-up
 $> 0 \Rightarrow$ above the x -axis

We need the x -values to the left of -2 and to the right of 1 .

$\{x : x < -2\} \cup \{x : x > 1\}$



E.g. 2 Solve the inequalities: (a) $3x^2 + 5x - 2 < 0$ (b) $10x^2 \geq x + 3$
N.B. For (b), feel free to use your calculator to find the roots.

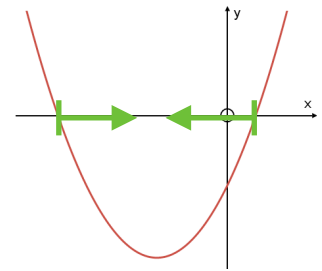
Working: (a) Solve $3x^2 + 5x - 2 = 0$ to find the roots:
 $(3x - 1)(x + 2) = 0$

Roots are $x = -2$ and $x = \frac{1}{3}$

Coefficient of x^2 is +ve so concave-up
 $< 0 \Rightarrow$ below the x -axis

We need the x -values **to the right of -2**
 and **to the left of $\frac{1}{3}$** .

$$\left\{ x : x > -2 \right\} \cap \left\{ x : x < \frac{1}{3} \right\}$$

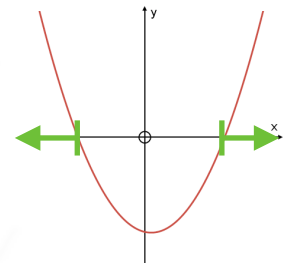


(b) Rearrange then solve $10x^2 - x - 3 \geq 0$
 By calculator, roots are $x = -\frac{1}{2}$ and $x = \frac{3}{5}$

Coefficient of x^2 is +ve so concave-up
 $\geq 0 \Rightarrow$ above the x -axis

We need the x -values **to the left of $-\frac{1}{2}$**
 and **to the right of $\frac{3}{5}$** .

$$\left\{ x : x \leq -\frac{1}{2} \right\} \cup \left\{ x : x \geq \frac{3}{5} \right\}$$



Unknown in denominator

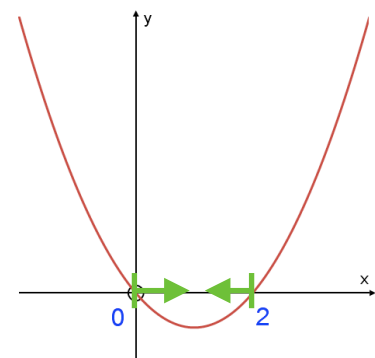
When the unknown is in the denominator (e.g. $\frac{2}{x} < 1$), we cannot simply multiply by x . This is because x could be positive or negative. Multiplying by x^2 causes no ambiguity, since $x^2 \geq 0$.

E.g. 3 Solve $\frac{2}{x} \geq 1$.

Working: Multiplying by x^2 : $2x \geq x^2$
 Rearranging: $x^2 - 2x \leq 0$
 Solving: $x(x - 2) \leq 0$
 \therefore the roots are $x = 0$ or $x = 2$

Coefficient of x^2 is +ve so concave-up
 $\leq 0 \Rightarrow$ below the x -axis

We need the x -values **to the right of 0**
 and **to the left of 2**.
 $\{x : x \geq 0\} \cap \{x : x \leq 2\}$

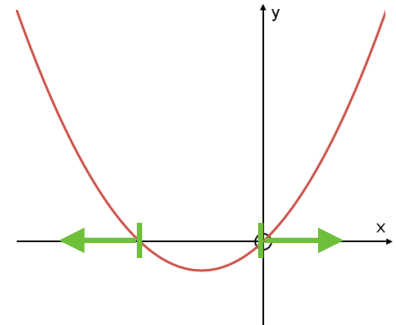


E.g. 4 Solve $\frac{1}{x} + 3 > 2$.

Working: Multiplying by x^2 : $x + 3x^2 > 2x^2$
 Rearranging: $x^2 + x > 0$
 Solving: $x(x + 1) > 0$
 \therefore the roots are $x = -1$ or $x = 0$

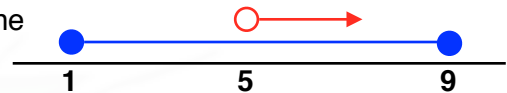
Coefficient of x^2 is +ve so concave-up
 $> 0 \Rightarrow$ above the x -axis

We need the x -values **to the left of -1**
 and **to the right of 0** .
 $\{x : x < -1\} \cup \{x : x > 0\}$



E.g. 5 State the set of values that satisfies $x > 5$ and $1 \leq x \leq 9$.

Working: Draw both inequalities on a number line
 The solution is the overlap
 $5 < x \leq 9$
 $\{x : x > 5\} \cap \{x : x \leq 9\}$



E.g. 6 Find the values of x which satisfy both $4(3 - x) \geq 13 - 5x$ and $7x + 6 \geq 3x^2$.

Working: **Linear inequality:** $12 - 4x \geq 13 - 5x$
 $x \geq 1$

Quadratic: $7x + 6 \geq 3x^2 \Rightarrow 3x^2 - 7x - 6 \leq 0$
 Solve $3x^2 - 7x - 6 = 0$ to get the roots

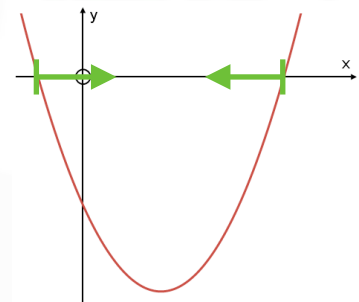
Roots are $x = -\frac{2}{3}$ and $x = 3$

Coefficient of x^2 is +ve so concave-up
 $\leq 0 \Rightarrow$ below the x -axis

We need the x -values **to the right of $-\frac{2}{3}$**
 and **to the left of 3** .

$$-\frac{2}{3} \leq x \leq 3$$

Overall: $\{x : x \geq 1\} \cap \{x : x \leq 3\}$



E.g. 7 A rectangular office is to be built measuring $(x - 9)$ metres wide and $(x - 6)$ metres long. Given that at least 28 m^2 of floor space is required, find the set of possible values of x .

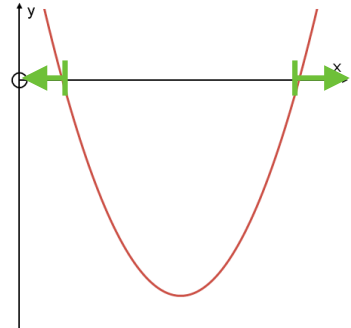
Working: $(x - 9)(x - 6) \geq 28$
Expanding and rearranging: $x^2 - 15x + 26 \geq 0$
Solve $x^2 - 15x + 26 = 0$
Roots are $x = 2$ and $x = 13$

Coefficient of x^2 is +ve so concave-up
 $\leq 0 \Rightarrow$ below the x -axis

We need the x -values **to the left of 2**
and **to the right of 13**.

$$x \leq 2 \text{ or } x \geq 13$$

$$\text{Since } x > 6, \{x : x \geq 13\}$$



Video: [Quadratic inequalities](#)

[Solutions to Starter and E.g.s](#)

Exercise

p44 3D Qu 1ace, 2ibd, 3-5 (2ic = 3)