Simplifying before Differentiating

Starter

1. **(Review of last lesson)**

Find the first derivative of: (a) $f(x) = 7x^{\frac{3}{8}}$ (b) $f(x) = -4x^{-\frac{12}{5}}$

Working: (a) $f'(x) = 7 \times \frac{3}{8}x^{\frac{3}{8}-1} = \frac{21}{8}x^{-\frac{5}{8}}$

(b) $f'(x) = (-4) \times \left(-\frac{12}{5}\right)x^{-\frac{12}{5}-1} = \frac{48}{5}x^{-\frac{17}{5}}$

2. Rewrite the following in the form kx^n:

(a) $\sqrt{x}$ (b) $\sqrt[3]{x^5}$ (c) $\frac{5}{x^7}$ (d) $\frac{8}{3\sqrt[4]{x^9}}$

Working: (a) $x^{\frac{1}{2}}$ (b) $x^{\frac{5}{3}}$

(c) $5x^{-7}$ (d) $\frac{8x^{-\frac{9}{4}}}{3}$

E.g. 1 Differentiate these functions:

(a) $y = \frac{1}{x}$ (b) $f(x) = -\frac{5}{x^7}$ (c) $y = \frac{9}{4x^5}$

Working: (a) $y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -1x^{-2} = -\frac{1}{x^2}$

(b) $f(x) = -\frac{5}{x^7} = -5x^{-7}$

$f'(x) = 35x^{-8} = \frac{35}{x^8}$

(c) $y = \frac{9}{4x^5} = \frac{9x^{-5}}{4}$

$\frac{dy}{dx} = -\frac{45x^{-6}}{4} = -\frac{45}{4x^6}$

E.g. 2 Differentiate these functions:

(a) $y = \sqrt{x^3}$ (b) $f(x) = \sqrt[7]{x^4}$ (c) $y = \dfrac{8}{\sqrt{x^5}}$

Working:

(a) $y = \sqrt{x^3} = x^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{3x^{\frac{1}{2}}}{2} = \frac{3\sqrt{x}}{2}$$

(b) $f(x) = \sqrt[7]{x^4} = x^{\frac{4}{7}}$

$$f'(x) = \frac{4x^{-\frac{3}{7}}}{7} = \frac{4}{7x^{\frac{3}{7}}} = \frac{4}{7\sqrt[7]{x^3}}$$

(c) $y = \dfrac{8}{\sqrt{x^5}} = 8x^{-\frac{5}{2}}$

$$\frac{dy}{dx} = -\frac{40x^{-\frac{7}{2}}}{2} = -\frac{20}{x^{\frac{7}{2}}} = -\frac{20}{\sqrt{x^7}}$$

E.g. 3 Differentiate these functions:

(a) $y = (3x-5)^2$ (b) $y = \dfrac{x^3-4}{3x}$ (c) $y = \dfrac{9x^5-7}{x^3}$

Working:

(a) $y = (3x-5)^2 = 9x^2 - 30x + 25$

$$\frac{dy}{dx} = 18x - 30$$

(b) $y = \dfrac{x^3-4}{3x} = \dfrac{x^3}{3x} - \dfrac{4}{3x} = \dfrac{x^2}{3} - \dfrac{4x^{-1}}{3}$

$$\frac{dy}{dx} = \frac{2x}{3} + \frac{4x^{-2}}{3} = \frac{2x}{3} + \frac{4}{3x^2}$$

(c) $y = \dfrac{9x^5-7}{x^3} = \dfrac{9x^5}{x^3} - \dfrac{7}{x^3} = 9x^2 - 7x^{-3}$

$$\frac{dy}{dx} = 18x + 21x^{-4} = 18x + \frac{21}{x^4}$$

Video: Differentiating polynomials
Video: Differentiating polynomials EQ

Solutions to Starter and E.g.s

Exercise
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