

## Simplifying before Integrating

### Starter

1. (Review of last lesson) Find: (a)  $\int x^{\frac{3}{4}} dx$  (b)  $\int x^{-\frac{2}{7}} dx$

**Working:** (a)  $\int x^{\frac{3}{4}} dx = \frac{1}{\frac{7}{4}} x^{\frac{3}{4}+1} + c = \frac{4}{7} x^{\frac{7}{4}} + c$

(b)  $\int x^{-\frac{2}{7}} dx = \frac{1}{\frac{5}{7}} x^{\frac{-2}{7}+1} + c = \frac{7}{5} x^{\frac{5}{7}} + c$

**E.g. 1** Find: (a)  $\int \frac{2}{x^3} dx$  (b)  $\int \frac{1}{7x^5} dx$  (c)  $\int 5\sqrt{x} dx$  (d)  $\int 4x\sqrt{x} dx$

**Working:** (a)  $\int \frac{2}{x^3} dx = \int 2x^{-3} dx = \frac{2x^{-2}}{-2} + c = -\frac{1}{x^2} + c$

(b)  $\int \frac{1}{7x^5} dx = \int \frac{x^{-5}}{7} dx = \frac{x^{-4}}{7 \times (-4)} + c = -\frac{1}{28x^4} + c$

(c)  $\int 5\sqrt{x} dx = \int 5x^{\frac{1}{2}} dx = \frac{10}{3} x^{\frac{3}{2}} + c = \frac{10}{3} \sqrt{x^3} + c$

(d)  $\int 4x\sqrt{x} dx = \int 4x^{\frac{3}{2}} dx = \frac{8}{5} x^{\frac{5}{2}} + c = \frac{8}{5} \sqrt{x^5} + c$

**E.g. 2** Find: (a)  $\int 4x(x^2 - 1) dx$  (b)  $\int \left(2x - \frac{1}{x}\right)^2 dx$

**Working:** (a)  $\int 4x(x^2 - 1) dx = \int (4x^3 - 4x) dx = x^4 - 2x^2 + c$

(b)  $\int \left(2x - \frac{1}{x}\right)^2 dx = \int \left(4x^2 - 4 + \frac{1}{x^2}\right) dx$   
 $= \int \left(4x^2 - 4 + x^{-2}\right) dx$   
 $= \frac{4}{3} x^3 - 4x - x^{-1} + c$   
 $= \frac{4}{3} x^3 - 4x - \frac{1}{x} + c$

**E.g. 3** Find: (a)  $\int \frac{3x^2}{\sqrt{x}} dx$  (b)  $\int \left( \frac{2+x}{x^5} \right) dx$

**Working:** (a)  $\int \frac{3x^2}{\sqrt{x}} dx = \int 3x^{\frac{3}{2}} dx = \frac{6}{5} x^{\frac{5}{2}} + c = \frac{6}{5} \sqrt{x^5} + c$

(b)  $\int \left( \frac{2+x}{x^5} \right) dx = \int \left( \frac{2}{x^5} + \frac{x}{x^5} \right) dx$   
 $= \int \left( 2x^{-5} + x^{-4} \right) dx$   
 $= \frac{2}{-4} x^{-4} + \frac{1}{-3} x^{-3} + c$   
 $= -\frac{1}{2x^4} - \frac{1}{3x^3} + c$

[Video: Integrating polynomials](#)

[Integrating polynomials EQ](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

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