Sine rule, including the ambiguous case

Starter

1. *(Review of previous material)* A triangle ABC is such that $AB = 10$ cm, $BC = 8$ cm and $\angle BAC = 20°$. Using the sine rule, find the length of AC to 3 s.f..

Working:

$$\frac{\sin A\hat{C}B}{10} = \frac{\sin 20°}{8}$$

$\sin A\hat{C}B \approx 0.4275$

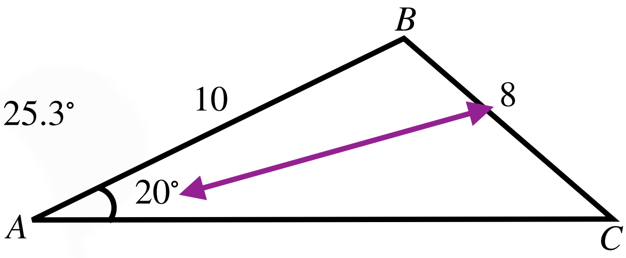
$\angle ACB \approx 25.3°$

$\Rightarrow \angle ABC \approx 180° - 20° - 25.3°$

$\approx 134.7°$

$$\frac{AC}{\sin 134.7} = \frac{8}{\sin 20}$$

$AC = 16.6$ (3 s.f.)



2. (a) By considering the graph of $y = \sin x$, or otherwise, find two angles for $0° \leq \theta \leq 180°$ which satisfy $\sin \theta = 0.4275$

(b) Hence find an alternative length for AB in the triangle from question 1.

(c) Sketch the two triangles.

Working:

(a) $\sin \theta = 0.4275 \quad \Rightarrow \quad \theta = \sin^{-1} 0.4275 = 25.3°$

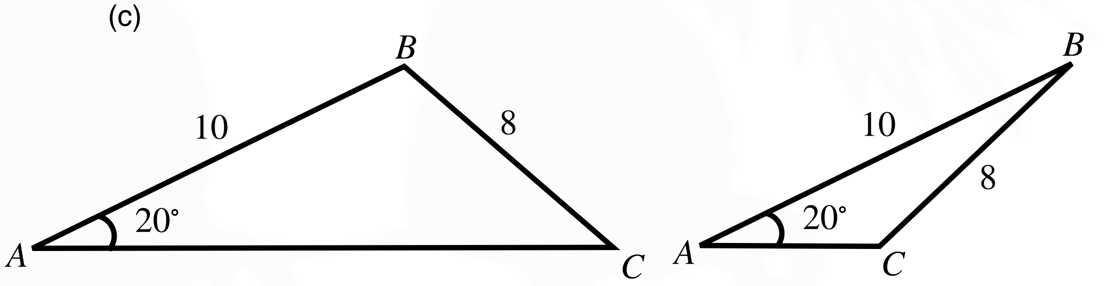
Since $\sin x \equiv \sin(180° - x)$: $\quad \theta = 180° - 25.3° = 154.7°$

(b) $\angle ABC \approx 180° - 20° - 154.7° \approx 5.31°$

$$\frac{AC}{\sin 5.31} = \frac{8}{\sin 20}$$

$AC = 2.16$ (3 s.f.)

(c)



E.g. 1 In the triangle XYZ, $\angle X = 29.5°$, $XY = 21$ cm and $YZ = 36$ cm. By calculation, decide whether the ambiguous case of the sine rule applies for this triangle.

Working:

$$\frac{\sin X\hat{Z}Y}{21} = \frac{\sin 29.5°}{36}$$

$\sin X\hat{Z}Y \approx 0.2872...$

$\angle Z \approx 16.7°$ or $\angle Z \approx 180° - 16.7° \approx 163°$

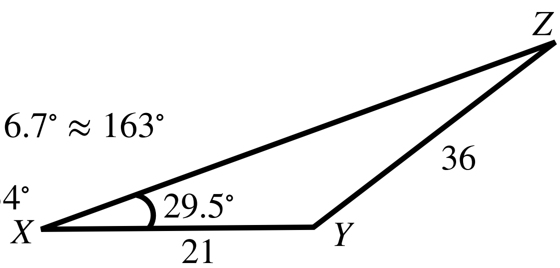
When $\angle Z \approx 16.7°$:

$\angle Y \approx 180° - 29.5° - 16.7° \approx 134°$

When $\angle Z \approx 163°$:

$\angle Y \approx 180° - 29.5° - 163° < 0°$

Therefore, there is no ambiguous case for this triangle.



Geometric understanding of the sine rule

E.g. 2 (a) Draw a horizontal line about 10 cm long and label the left end A.

(b) From A, draw a side of length 8 cm at an angle of about $45°$ above the horizontal. Label the end of this line B.

(c) Open your compass to 6.5 cm. Put your compass point on B and draw an arc that cuts the horizontal line in two places. Label these points C_1 and C_2.

(d) Draw dotted lines from B to C_1 and B to C_2.

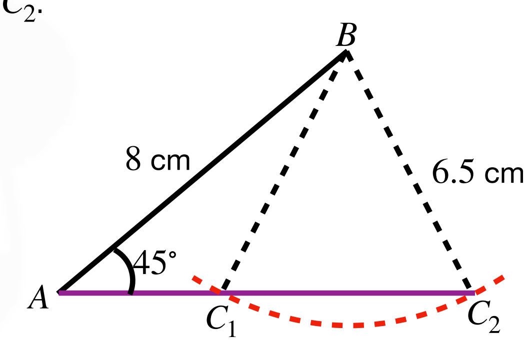
(e) Measure the lengths from A to the intersection points C_1 and C_2, giving your answers to the nearest millimetre.

(f) Measure the angles at $\angle ABC_1$ and $\angle ABC_2$.

Working: (a)-(d)

(e) $AC_1 = 2.4$ cm
$AC_2 = 8.8$ cm

(f) $\angle ABC_1 \approx 15°$
$\angle ABC_2 \approx 74°$



Geogebra 1: Sine rule ambiguous case
Geogebra 2: Sine rule ambiguous case animation

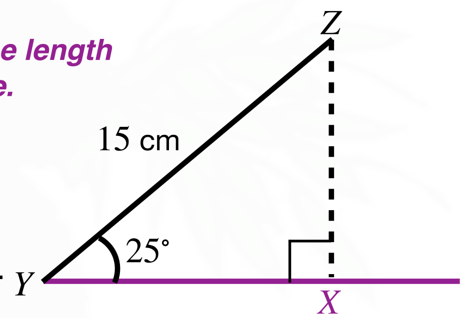
E.g. 3 In the triangle XYZ, $YZ = 15$ and $\angle XYZ = 25°$. Find the range of values of the length XZ, with $XZ < 15$ cm such that the ambiguous case exists for the triangle XYZ.

Working: ***The length XZ must be greater than the length required to form a right-angled triangle.***

$\sin 25° = \frac{XZ}{15}$

$XZ = 15 \sin 25°$
$= 6.34$

The range of values is $6.34 < XZ < 15$.



Video: Sine rule
Video: Sine rule - ambiguous case

Exam questions: Sine rule

Solutions to Starter and E.g.s

Exercise
p207 11A Qu 1i, 2i, 3, 4-5, (6-7 red)