Solving Exponential Equations

Starter

1. **(Review of last lesson)** Write these as a single logarithm:
(a) $5\log_a 2 - 2\log_a 4 + 3\log_a 3$ (b) $2\log_a 2 - (\log_a 5 + \log_a 8)$

Working: (a)
$$\begin{aligned} 5\log_a 2 - 2\log_a 4 + 3\log_a 3 &= \log_a 2^5 - \log_a 4^2 + \log_a 3^3 \\ &= \log_a 32 - \log_a 16 + \log_a 27 \\ &= \log_a \frac{32}{16} + \log_a 27 \\ &= \log_a\left(\frac{32}{16} \times 27\right) \\ &= \log_a 54 \end{aligned}$$

(b)
$$\begin{aligned} 2\log_a 2 - (\log_a 5 + \log_a 8) &= 2\log_a 2 - (\log_a 40) \\ &= \log_a 2^2 - \log_a 40 \\ &= \log_a \frac{4}{40} \\ &= \log_a \frac{1}{10} \end{aligned}$$

2. **(Review of last lesson)** Given that a and b are positive constants, and $a > b$, solve the simultaneous equations: $a + b = 13$ and $\log_6 a + \log_6 b = 2$.

Working: $\log_6 a + \log_6 b = 2 \Rightarrow \log_6 ab = 2$

$ab = 6^2 \Rightarrow a = \dfrac{36}{b}$

From $a + b = 13$: $\dfrac{36}{b} + b = 13$

Multiply by b and rearrange: $b^2 - 13b + 36 = 0$
$(b-9)(b-4) = 0$

So $b = 4$ or $b = 9$
When $b = 4$, $a = 9$
When $b = 9$, $a = 4$
Since $a > b$, $a = 9$, $b = 4$

E.g. 1 Solve (a) $7^x = 2$ (b) $2^{3x-1} = 5$ (c) $11^{6x} = 10^{90}$
Give your answer exactly (i.e. in logs in its simplest form) and to 3 s.f.

Working: (a) Take logs of both sides:
$$\log 7^x = \log 2$$
$$x\log 7 = \log 2 \quad \textit{(3rd law)}$$
$$x = \frac{\log 2}{\log 7} = 0.356 \text{ (3 s.f.)}$$

(b) Take logs of both sides: $\log 2^{3x-1} = \log 5$

$(3x-1)\log 2 = \log 5$ *(3rd law)*

Expand the brackets: $3x\log 2 - \log 2 = \log 5$

Rearrange: $3x\log 2 = \log 5 + \log 2$

$$x = \frac{\log 5 + \log 2}{3\log 2}$$

$$x = \frac{\log 10}{\log 8} \quad \textit{(1st \& 3rd law)}$$

$$= \frac{1}{\log 8} = 1.11 \text{ (3 s.f.)}$$

Remember: $\log 10 \equiv \log_{10} 10 = 1$

(c) Take logs of both sides: $\log 11^{6x} = \log 10^{90}$

$6x\log 11 = 90\log 10$ *(3rd law)*

$$x = \frac{90\log 10}{6\log 11}$$

$$x = \frac{15}{\log 11} = 14.4 \text{ (3 s.f.)}$$

Remember: $\log 10 \equiv \log_{10} 10 = 1$

E.g. 2 Solve $8 \times 5^x = 2$, giving your answer exactly and to 3 s.f.

Working: $8 \times 5^x = 2 \quad \Rightarrow \quad 5^x = \frac{1}{4}$

Take logs of both sides: $\log 5^x = \log \frac{1}{4}$

Use 3rd law of logs: $x\log 5 = \log \frac{1}{4}$

Rearrange: $x = \dfrac{\log \frac{1}{4}}{\log 5} = -0.861$

E.g. 3 Solve $6 \times 7^{2x-1} = 23$, giving your answer to 3 s.f..

Working: $6 \times 7^{2x-1} = 23 \quad \Rightarrow \quad 7^{2x-1} = \frac{23}{6}$

Take logs of both sides: $\log 7^{2x-1} = \log \frac{23}{6}$

Use 3rd and 2nd law of logs: $(2x-1)\log 7 = \log 23 - \log 6$

Expand the brackets: $2x\log 7 - \log 7 = \log 23 - \log 6$

Rearrange: $2x\log 7 = \log 23 - \log 6 + \log 7$

$$x = \frac{\log 23 - \log 6 + \log 7}{2\log 7}$$

$$x = 0.845 \text{ (3 s.f.)}$$

N.B. $\log \frac{23}{6}$ could be left as it was.

E.g. 4 Solve $11 \times 7^{x+1} = 3^{x+2}$, giving your answer exactly and to 3 s.f..

Working: Dividing by 11 doesn't really help so start with taking logs…

Take logs of both sides: $\log\left(11 \times 7^{x+1}\right) = \log 3^{x+2}$

1st law of logs: $\log 11 + \log 7^{x+1} = \log 3^{x+2}$

3rd law of logs: $\log 11 + (x+1)\log 7 = (x+2)\log 3$

Expand: $\log 11 + x\log 7 + \log 7 = x\log 3 + 2\log 3$

Collect like terms: $x\log 7 - x\log 3 = 2\log 3 - \log 11 - \log 7$

Factorise: $x(\log 7 - \log 3) = 2\log 3 - \log 11 - \log 7$

Exact answer: $x = \dfrac{2\log 3 - \log 11 - \log 7}{\log 7 - \log 3}$

$x = -2.53$ (3 s.f.)

Video: Exponential and log equations

Video: Solving inequalities involving logs

Solutions to Starter and E.g.s

Exercise

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