Solving Problems in Kinematics

Starter

1. A particle moving in a straight line passes the point A when $t = 0$. Its velocity, v m/s, satisfies: $v = \begin{cases} t(3-t) & \text{for } 0 \le t \le 3 \\ 6-2t & \text{for } t > 3 \end{cases}$

where t is measured in seconds.

(a) Find the distance travelled when the particle first comes to rest for $t > 0$.

(b) Find the displacement after 4 seconds.

(c) Find the time at which the particle returns to A.

Working:

(a) Particle at rest $\Rightarrow v = 0$ so $t(3-t) = 0$

Since $t > 0$, $t = 3$

So $\int_0^3 t(3-t)dt = \int_0^3 (3t - t^2)dt$

$= \left[\frac{3}{2}t^2 - \frac{1}{3}t^3\right]_0^3$

$= \left(\frac{3}{2} \times 3^2 - \frac{1}{3} \times 3^3\right) - \left(0 - 0\right)$

$= 4.5$ m

(b) Between 3 s and 4 s the particle is moving in the opposite direction.

$\int_3^4 (6-2t)dt = \left[6t - t^2\right]_3^4$

$= \left(6 \times 4 - 4^2\right) - \left(6 \times 3 - 3^2\right) = -1$

So displacement $= 4.5 - 1 = 3.5$ m

(c) Since the value of $\int_3^T (6-2t)dt$ will be negative, we need to solve

$\int_3^T (6-2t)dt = -4.5$

$\left[6t - t^2\right]_3^T = -4.5$

$(6T - T^2) - (18 - 9) = -4.5$

$T^2 - 6T + 4.5 = 0$

$T = 5.12$ or $T = 0.879$

The particle returns to A after 5.12 seconds

Solutions to Starter and E.g.s

Exercise

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