

Solving Quadratic Equations

Starter

1. **(Review of last lesson)** Rationalise the denominator of $\frac{3\sqrt{2}}{5 + \sqrt{2}}$.

Working:

$$\frac{3\sqrt{2}}{5 + \sqrt{2}} = \frac{3\sqrt{2}}{5 + \sqrt{2}} \times \frac{5 - \sqrt{2}}{5 - \sqrt{2}}$$

$$= \frac{3\sqrt{2}(5 - \sqrt{2})}{23}$$

2. Solve the equation $x^2 - 5x - 2 = 0$ using your Classwiz calculator. The method is below:

1. Press Menu (top row)
2. Select ' $x y = 0$ ' by highlighting it and pressing '=' (Use the arrow buttons to scroll down/across)
3. Press 2 (Polynomial)
4. Press 2 (Degree, since it is a quadratic - power of 2)
5. (Enter data)
6. Press '=' to solve

Video: [Classwiz - solving quadratic equations](#)

Working: $x = \frac{5 \pm \sqrt{33}}{2} = 5.37, -0.372$

3. **(Review of GCSE material)** Solve by factorising $2x^2 + 7x + 3 = 0$.

Working:

$$2 \times 3 = 6 \quad \text{Multiply: } 6 = 6 \times 1$$

$$\text{Add: } 7 = 6 + 1$$

$$2x^2 + 6x + x + 3 = 0$$

$$2x(x + 3) + 1(x + 3) = 0$$

$$(x + 3)(2x + 1) = 0 \quad \text{other methods also acceptable}$$

$$x + 3 = 0 \quad \text{or} \quad 2x + 1 = 0$$

$$x = -3 \quad \text{or} \quad x = -\frac{1}{2}$$

4. Factorise $4 + 3x - x^2$.

Working:

Either: By inspection: $4 + 3x - x^2 = (4 - x)(1 + x)$ or

Or: Multiply $4 + 3x - x^2$ by (-1) : $x^2 - 3x - 4$

Factorise: $x^2 - 3x - 4$: $(x - 4)(x + 1)$

Multiply by -1 : $-1 \times (x - 4)(x + 1) = (4 - x)(x + 1)$

- E.g. 1** Factorise $35 - 2x - x^2$.

Working:

Multiply by (-1) and reorder: $x^2 + 2x - 35$

Factorise as normal: $(x - 5)(x + 7)$

Change the signs in the bracket with the $-$ sign:

$(5 - x)(7 + x)$ or $(5 - x)(x + 7)$

E.g. 2 Solve $12 + x - x^2 = 0$.

Working: $12 + x - x^2 = 0$ is the same as $x^2 - x - 12 = 0$
 $(x - 4)(x + 3) = 0$
 $x - 4 = 0$ or $x + 3 = 0$
 $x = 4$ or $x = -3$

E.g. 3 Solve the equation $5x = \frac{1}{x} - 9$ giving your answers to 3 s.f. and showing all your working.

Working: Multiply the equation by x : $5x^2 = 1 - 9x$
Rearrange so “= 0”: $5x^2 + 9x - 1 = 0$
Answers to 3 s.f. suggests the quadratic won't factorise and since the questions requires all working shown, use the quadratic formula.
 $a = 5, b = 9, c = -1$

$$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 5 \times (-1)}}{2 \times 5}$$

$$x = \frac{-9 \pm \sqrt{101}}{10} \quad \text{so} \quad x = 0.105 \text{ or } x = -1.90$$

E.g. 4 The trapezium has area 50 units². Find the exact value of the height of the trapezium.

N.B. Area of trapezium = $\frac{1}{2}(a + b)h$

Working: Area of trapezium = $\frac{1}{2}(a + b)h$

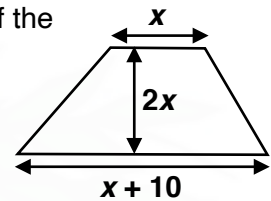
$$\frac{1}{2}(x + x + 10) \times 2x = 50$$

$$x(2x + 10) = 50$$

$$2x^2 + 10x - 50 = 0$$

$$\text{By calculator: } x = \frac{-5 \pm 5\sqrt{5}}{2}$$

Since $x > 0$, height of trapezium is $2x = 5(\sqrt{5} - 1)$



E.g. 5 A cylindrical tin of height 6 cm has total surface area of 54π cm². Without using a calculator, find the exact value of the volume of the tin.

N.B. Surface area of a cylinder = $2\pi r^2 + 2\pi rh$

Volume of cylinder = $\pi r^2 h$

“Exact” in this case means leave your answer in terms of π

Working: Substituting: $2\pi r^2 + 2\pi r \times 6 = 54\pi$

$$\text{Dividing by } 2\pi \text{ and rearranging: } r^2 + 6r - 27 = 0$$

$$(r - 3)(r + 9) = 0$$

$$r = 3 \text{ or } r = -9$$

Since $r > 0$, $r = 3$

$$\text{Substituting: } \text{Volume} = \pi \times 6 \times 3^2 = 54\pi \text{ cm}^3$$

Exercise

p30 3A Qu 1i, 2i, 3-8

