Standard deviation

Starter

1. Consider the data values: $2, 3, 7, 4, 9, 6, 4$.
   (a) Calculate the mean, $\bar{x}$, for this data.
   (b) Write down the individual deviations of the data values from the mean i.e. $x - \bar{x}$.
   (c) Find the sum of these deviations. What does this number tell us?

Working:

(a) Mean, $\bar{x} = \dfrac{2+3+7+4+9+6+4}{7} = 5$

(b) The individual deviations are: $2-5=-3$
$3-5=-2$ $\quad 7-5=2$ $\quad 4-5=-1$
$9-5=4$ $\quad 6-5=1$ $\quad 4-5=-1$

(c) Sum of deviations $= (-3)+(-2)+2+(-1)+4+1+(-1)$
$= 0$
This numbers tells us exactly nothing! The sum of the deviations from the mean will always be zero.

2. Find the interquartile range for these data values:
   (a) $8, 14, 1, 9, 11, 5, 2,$
   (b) $18, 23, 27, 15, 22, 26$

Working:

(a) Put in ascending order: $1, 2, 5, 8, 9, 11, 14$
Locate ***median*** & bracket the two halves: $(1, 2, 5), 8, (9, 11, 14)$
Find the middle number for each bracket:
Lower quartile, $Q_1 = 2$ Upper quartile, $Q_3 = 11$
Interquartile range, $IQR = 11 - 2 = 9$

(b) Put in ascending order: $15, 18, 22, 23, 26, 27$
Since there is an ***even number*** of data values:
Bracket the two halves: $(15, 18, 22), (23, 26, 27)$
Find the middle number for each bracket:
Lower quartile, $Q_1 = 18$ Upper quartile, $Q_3 = 26$
Interquartile range, $IQR = 26 - 18 = 8$

E.g. 1 Find the mean and standard deviation of this set of data: $35, 31, 27, 34, 23$

Working:

Mean, $\bar{x} = \dfrac{35+31+27+34+23}{5} = 30$

$\Sigma x^2 = 35^2 + 31^2 + 27^2 + 34^2 + 23^2 = 4600$

Standard deviation, $\sigma = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\dfrac{4600}{5} - 30^2} = 2\sqrt{5} \approx 4.47$

E.g. 2 These are the scores of two dart players:

Maxine: $100, 60, 85, 41, 28, 46$

Jeremy: $180, 30, 26, 140, 26, 60$

(a) Calculate the mean and standard deviation of each player.

(b) Use your answers to (a) to compare the two players.

Working: (a) Means:

$$\text{Maxine, } \bar{x}_M = \frac{100 + 60 + 85 + 41 + 28 + 46}{6} = 60$$

$$\text{Jeremy, } \bar{x}_J = \frac{180 + 30 + 26 + 140 + 26 + 60}{6} = 77$$

Standard deviations:

$$\sigma_M = \sqrt{\frac{100^2 + 60^2 + 85^2 + 41^2 + 28^2 + 46^2}{6} - 60^2} \approx 25.2$$

$$\sigma_J = \sqrt{\frac{180^2 + 30^2 + 26^2 + 140^2 + 26^2 + 60^2}{6} - 77^2} \approx 60.9$$

(b) Jeremy has a higher average (77 vs. 60) but he is less consistent since his standard deviation is higher (60.9 vs. 25.2).

E.g. 3 A data set of 8 values has a sum of 264 and the sum of the squares is 12593. Find the mean and the standard deviation.

Working:

$$\bar{x} = \frac{264}{8} = 33$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{12593}{8} - 33^2} = 22.0 \text{ (3 s.f.)}$$

Effect on standard deviation of adding a number to each data value

E.g. 4 From ***E.g. 1***, the mean and standard deviation of the data values $35, 31, 27, 34, 23$ are 30 and 4.47 respectively.

(a) Add 7 to each data value. Calculate the mean and standard deviation. What do you notice?

(b) Multiply each original data value by 3. Calculate the mean and standard deviation and write down what you notice.

Working:

(a) The new data values are $42, 38, 34, 41, 30$.

$$\text{Mean, } \bar{x} = \frac{42 + 38 + 34 + 41 + 30}{5} = 37$$

$$\Sigma x^2 = 42^2 + 38^2 + 34^2 + 41^2 + 30^2 = 6945$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{6945}{5} - 37^2} = 2\sqrt{5} \approx 4.47$$

The mean has increased by the number added, but adding a number did not affect the standard deviation.

(b) The new data values are $105, 93, 81, 102, 69$.

$$\text{Mean, } \bar{x} = \frac{105 + 93 + 81 + 102 + 69}{5} = 90$$

$$\Sigma x^2 = 105^2 + 93^2 + 81^2 + 102^2 + 69^2 = 41400$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{41400}{5} - 90^2} = 6\sqrt{5} \approx 13.4$$

Both the mean and the standard deviation have been multiplied by the number that each of the data values have been multiplied by.

E.g. 5 Use your calculator to find the mean and standard deviation of this data set:

$$84, 96, 61, 89, 63, 95, 82, 70, 48, 63, 97, 85, 32, 56$$

Working:

Mean, $\bar{x} = 72.9$ (3 s.f.)

Standard deviation, $\sigma = 19.2$ (3 s.f.) (σx)

Video: Standard deviation

Exam questions: Standard deviation with discrete data

Exam questions: Standard deviation with continuous data

Solutions to Starter and E.g.s

Exercise

p336 16B Qu 1i, 2i, 3-7, (8-9 red)