Stationary Points

Starter

1. **(Review of last lesson)** A curve has equation $y = x^3 - px + q$. The tangent to this curve at the point $(2, -8)$ is parallel to the $x-$axis.
   (a) Find the values of p and q.
   (b) Find the coordinates of the other point where the tangent is parallel to the $x-$axis.

Working:

(a) At (2, −8): $-8 = 2^3 - 2p + q \Rightarrow 2p - q = 16$

Tangent parallel to the $x-$axis $\Rightarrow \frac{dy}{dx} = 0$ when $x = 2$

$\frac{dy}{dx} = 3x^2 - p$

When $x = 2$, $3 \times 2^2 - p = 0 \Rightarrow p = 12$

Substitute into $2p - q = 16$: $24 - q = 16$

$q = 8$

(b) $y = x^3 - 12x + 8$

$\frac{dy}{dx} = 3x^2 - 12$

Parallel to the $x-$axis $\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 3x^2 - 12 = 0$

$x^2 = 4 \quad \therefore x = \pm 2$

When $x = -2$, $y = (-2)^3 - 12 \times (-2) + 8 = 24$

The other point is $(-2, 24)$

N.B. A ***stationary point*** is where the ***gradient is zero***.

2. Find the stationary points of the curve $y = x^3 - 15x^2 + 48x + 7$.

Working:

$\frac{dy}{dx} = 3x^2 - 30x + 48$

A stationary point occurs when $\frac{dy}{dx} = 0$ so $3x^2 - 30x + 48 = 0$

$x^2 - 10x + 16 = 0 \Rightarrow (x - 8)(x - 2) = 0$

$x = 2$ or $x = 8$

When $x = 2$, $y = 2^3 - 15 \times 2^2 + 48 \times 2 + 7 = 51$

When $x = 8$, $y = 8^3 - 15 \times 8^2 + 48 \times 8 + 7 = -57$

The stationary points are at $(2, 51)$ and $(8, -57)$

E.g. 1 Find the stationary point(s) of the curve $y = \dfrac{x^2 + 9}{2x}$.

Working:

$$y = \frac{x^2 + 9}{2x} = \frac{x^2}{2x} + \frac{9}{2x} = \frac{x}{2} + \frac{9x^{-1}}{2}$$

$$\frac{dy}{dx} = \frac{1}{2} - \frac{9x^{-2}}{2} = \frac{1}{2} - \frac{9}{2x^2}$$

A stationary point occurs when $\dfrac{dy}{dx} = 0$ so $\dfrac{1}{2} - \dfrac{9}{2x^2} = 0$

$$\frac{1}{2} = \frac{9}{2x^2} \quad \Rightarrow \quad x^2 = 9 \quad \Rightarrow \quad x = \pm 3$$

When $x = 3$, $y = \dfrac{3^2 + 9}{2 \times 3} = 3$

When $x = 3$, $y = \dfrac{(-3)^2 + 9}{2 \times (-3)} = -3$

The stationary points are at $(3, 3)$ and $(-3, -3)$.

E.g. 2 The curve $y = 3x^4 + ax^3 + bx^2$ passes through the point $(1, 3)$ and has a stationary point when $x = 2$. Find a and b.

Working:

Passes through the point (1, 3): $3 = 3 + a + b \Rightarrow a + b = 0$

$$\frac{dy}{dx} = 12x^3 + 3ax^2 + 2bx$$

When $x = 2$, $\dfrac{dy}{dx} = 0$: $12 \times 2^3 + 3a \times 2^2 + 2b \times 2 = 0$

So $12a + 4b = -96 \Rightarrow 3a + b = -24$

Since $b = -a \Rightarrow 2a = -24 \therefore a = -12$

If $a = -12$, $b = 12$

Video: Stationary points

Solutions to Starter and E.g.s

Exercise

p279 14B Qu 1 (just find the stationary points)