

## Surds

### Starter

1. (Review of last lesson)

Without a calculator, evaluate: (a)  $32^{\frac{2}{5}}$  (b)  $64^{-\frac{5}{6}}$

**Working:** (a)  $32^{\frac{2}{5}} = (32^{\frac{1}{5}})^2 = 2^2 = 4$

(b)  $64^{-\frac{5}{6}} = \frac{1}{64^{\frac{5}{6}}} = \frac{1}{(\sqrt[6]{64})^5} = \frac{1}{2^5} = \frac{1}{32}$

2. (Review of last lesson) Find integers  $x$  and  $y$  such that  $4^x \times 3^y = 24^8$ .

**Working:**  $24^8 = (4 \times 2 \times 3)^8$   
 $= 4^8 \times 2^8 \times 3^8$   
 $= 4^8 \times 4^4 \times 3^8$   
 $= 4^{12} \times 3^8$   
 $x = 12, y = 8$

3. (Review of GCSE material)

Simplify: (a)  $\frac{\sqrt{28}}{4}$  (b)  $(4 + \sqrt{5})(3 - \sqrt{5})$   
 (c)  $\frac{4}{\sqrt{2}}$  (d)  $\frac{4}{2 + \sqrt{3}}$

**Working:** (a)  $\sqrt{28} = \sqrt{4 \times 7} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$

(b)  $(4 + \sqrt{5})(3 - \sqrt{5}) = 12 - 5\sqrt{5} + 3\sqrt{5} - 5$   
 $= 7 - \sqrt{5}$

(c)  $\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$   $\sqrt{x} \times \sqrt{x} = x$

(d)  $\frac{4}{2 + \sqrt{3}} = \frac{4}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$   
 $= \frac{4(2 - \sqrt{3})}{4 - 3}$  *the  $-2\sqrt{3}$  and  $2\sqrt{3}$  terms cancel*  
 $= 4(2 - \sqrt{3})$

**E.g. 1** Simplify  $\sqrt{48}$ .

**Working:**  $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$   
 Note that 16 is the highest square number so is the best factor.  
 4 could also be used but requires 2 bites at the cherry  
 $\sqrt{48} = \sqrt{4 \times 12} = \sqrt{4} \times \sqrt{12} = 2\sqrt{12}$   
 Then  $2\sqrt{12} = 2 \times \sqrt{4 \times 3} = 2 \times \sqrt{4} \times \sqrt{3} = 2 \times 2 \times \sqrt{3} = 4\sqrt{3}$

**E.g. 2** Simplify: (a)  $\frac{7}{\sqrt{3}}$  (b)  $\frac{18}{5\sqrt{6}}$

**Working:**

(a)  $\frac{7}{\sqrt{3}} = \frac{7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$   $\sqrt{3} \times \sqrt{3} = 3$

(b)  $\frac{18}{5\sqrt{6}} = \frac{18}{5\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{18\sqrt{6}}{5 \times 6} = \frac{3\sqrt{6}}{5}$   $\sqrt{6} \times \sqrt{6} = 6$

**E.g. 3** Simplify: (a)  $\frac{1}{\sqrt{7}-2}$  (b)  $\frac{2\sqrt{3}-1}{4-\sqrt{3}}$

**Working:**

(a)  $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$   
 $= \frac{\sqrt{7}+2}{7-4}$  *the  $2\sqrt{7}$  and  $-2\sqrt{7}$  terms cancel*  
 $= \frac{3}{\sqrt{7}+2}$

(b)  $\frac{2\sqrt{3}-1}{4-\sqrt{3}} = \frac{2\sqrt{3}-1}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}}$   
 $= \frac{8\sqrt{3}+6-4-\sqrt{3}}{16-3}$   *$4\sqrt{3}$  and  $-4\sqrt{3}$  cancel*  
 $= \frac{7\sqrt{3}+2}{13}$

**E.g. 4** Simplify: (a)  $\frac{8}{\sqrt{21}+\sqrt{7}}$  (b)\*  $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{5}-\sqrt{2})}$

**Working:**

(a)  $\frac{8}{\sqrt{21}+\sqrt{7}} = \frac{8}{\sqrt{21}+\sqrt{7}} \times \frac{\sqrt{21}-\sqrt{7}}{\sqrt{21}-\sqrt{7}}$   
 $= \frac{8(\sqrt{21}-\sqrt{7})}{21-7}$   
 $= \frac{4(\sqrt{21}-\sqrt{7})}{7}$

(b) It is a good idea to do this question in 2 stages:  
 $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{5}-\sqrt{2})} = \frac{\sqrt{2}}{\sqrt{15}-\sqrt{6}} \times \frac{\sqrt{15}+\sqrt{6}}{\sqrt{15}+\sqrt{6}}$   
 $= \frac{\sqrt{30}+\sqrt{12}}{15-6}$   
 $= \frac{\sqrt{30}+2\sqrt{3}}{9}$

**Exercise**

p22 2B Qu 1-4ac(i), 5-12

