Travel Graphs

Starter

1. **(Review of last lesson)** A car accelerates such that its velocity is given by $v = 14 + 0.45t^2 - 0.03t^3$. Find:
   (a) a formula for acceleration and
   (b) the speed of the car once the car stops accelerating.

Working: (a) $a = \dfrac{dv}{dt} = 0.9t - 0.09t^2$

(b) The car stops accelerating when $a = 0$: $0.9t - 0.09t^2 = 0$

$0.09t(10 - t) = 0$

$\therefore t = 0$ or $t = 10$

When $t = 10$, $v = 14 + 0.45 \times 10^2 - 0.03 \times 10^3$

$= 14 + 45 - 30$

$= 29$ m/s

2. **(Review of last lesson)** A hot air balloon is at a height of 200 m and descending at 6 m/s. The burner is operated for 15 seconds, providing an upward acceleration given by $a = (0.03t^2 - 0.002t^3)$ m/s² after t seconds.
   (a) How long does it take for the balloon to stop descending?
   (b) Find its height at this point.
   N.B. For (a), you will need to solve the equation using your calculator: Menu >> xy = 0 >> 2:Polynomial >> Degree 4.

Working: (a) We need to find the time when $v = 0$

$v = \int a\,dt = \int (0.03t^2 - 0.002t^3)dt = 0.01t^3 - 0.0005t^4 + c$

When $t = 0, v = -6 \quad \Rightarrow \quad c = -6$

$v = 0.01t^3 - 0.0005t^4 - 6$

$t = 17.9, t = 11.0$

Since the burner is operated for 15 s, $t = 11.0$

(b) $x = \int (0.01t^3 - 0.0005t^4 - 6)dt$

$= 0.0025t^4 - 0.0001t^5 - 6t + k$

When $t = 0, x = 200 \quad \Rightarrow \quad k = 200$

$x = 0.0025t^4 - 0.0001t^5 - 6t + 200$

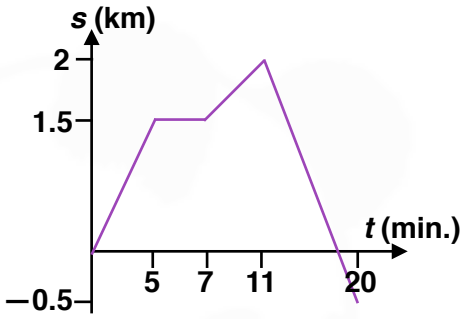
When $t = 11.0, x = 154.49$ m.

The balloon's height at this point is 154.49m

3. A girl goes for a run along a straight path. Her journey is detailed below:
   - She runs 1.5 km in 5 minutes, then rests for 2 minutes.
   - She then jogs 0.5 km in 4 minutes, in the same direction as before.
   - Finally she runs 2.5 km back in the same direction she came, passing her starting point along the way.
   - She finishes 20 minutes after she first set off.

   Show her journey on a displacement-time graph.

Working:



E.g. 1 A train takes 10 minutes to travel between two stations. The train accelerates from rest at a rate of 0.5 m/s² for 30 s. It then travels at a constant speed and is finally brought to rest in 15 s with a constant deceleration.

(a) Sketch a velocity-time graph for the journey.

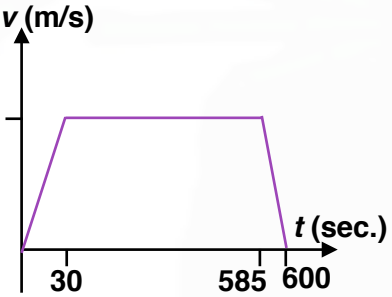
(b) Find

(i) the steady speed

(ii) the rate of deceleration and

(iii) the distance between the two stations.

Working: (a)



(b) (i) Velocity $= 0.5 \times 30 = 15$ m/s

(ii) Deceleration $= \dfrac{15}{15} = 1$ m/s²

(iii) Distance between stations = Area under graph

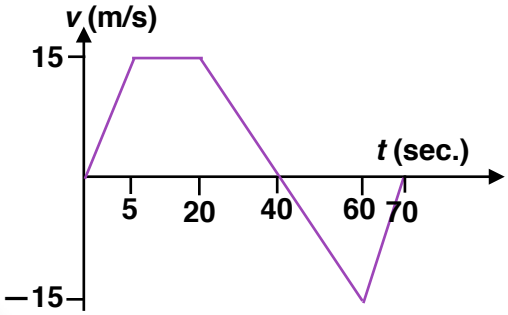
$$= \frac{1}{2}(600 + 555) \times 15$$
$$= 8662.5 \text{ m}$$

E.g. 2 The velocity time graph shows the motion of a particle moving in a straight line. Find:

(a) the magnitude of the particle's greatest acceleration during the motion

(b) the total distance travelled

(c) the total displacement of the particle



Working:

(a) The magnitude of the acceleration means we are not worried about whether the acceleration is positive or negative.

0–5 s: $a = 15 \div 5 = 3$

20–60 s: $a = 30 \div 40 = 0.75$

60–70 s: $a = 15 \div 10 = 1.5$

The greatest acceleration is 3 m/s^2

(b) Total distance travelled $\equiv$ Area between graph and the $t-$axis

Area above the $t-$axis $= \frac{1}{2}(15 + 40) \times 15 = 412.5$

Area below the $t-$axis $= \frac{1}{2} \times 30 \times 15 = 225$

Total distance travelled $= 412.5 + 225 = 637.5$ m

(c) Displacement means that area above the $t-$axis is positive but area below the $t-$axis is negative.

$\therefore$Total displacement travelled $= 412.5 - 225 = 187.5$ m

Video: Displacement-time graphs

Video: Velocity-time graphs

Velocity-time graphs EQ

Solutions to Starter and E.g.s

Exercise

p438 19D Qu 1, 3, 4ab, 5-8