Vertical Motion under Gravity

Starter

1. **(Review of last lesson)**
A cyclist starts riding up a hill with velocity 8 m/s. At the top of the hill, which is 96 m long, the velocity is 4 m/s. Assuming constant acceleration, find its value.

Working: $u = 8, s = 96, v = 4, a = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $4^2 = 8^2 + 2 \times a \times 96$

$-48 = 192a$

$a = -\frac{1}{4}$

The acceleration is $-\frac{1}{4}$ m/s.

2. **(Review of last lesson)** A stolen car, travelling at a constant speed of 40 m/s, passes a police car parked in a lay-by. The police car sets off three seconds later, accelerating uniformly at 8 m/s^2.

(a) How long does the police car take to intercept the stolen vehicle?

(b) How far from the lay-by, to the nearest metre, does the interception take place?

Working: (a) ***Both cars travel the same distance but the police car will travel for 3 seconds less time.***

Police car: $u = 0, a = 8, s = S, t = T$

Stolen car: $u = 40, v = 40, a = 0, s = S, t = T + 3$

Police: No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $S = 0 + \frac{1}{2} \times 8 \times T^2$

$S = 4T^2$

Stolen: $s = ut + \frac{1}{2}at^2$: $S = 40(T + 3) + 0$

$S = 40(T + 3)$

Both cars travel the same distance: $4T^2 = 40(T + 3)$

$T^2 - 10T - 30 = 0$

$T = 5 \pm \sqrt{55}$

Since $T \geq 0$, the times taken to intercept is $5 + \sqrt{55} \approx 12.4$ s

(b) Police car: $S = 4T^2 = 4(5 + \sqrt{55})^2 \approx 616.65$

The interception take place 617 m from the lay-by.

E.g. 1 (a) A ball is thrown upwards into the air. If it takes 5 seconds to reach the top point, how long will it take to drop down to the point from which it was thrown.

(b) What can you say about the velocity at the highest point?

Working: (a) 5 seconds, it takes the same time to come down as it takes to go up.

(b) Velocity $= 0$

E.g. 2 A stone dropped from the top of a cliff takes 5 seconds to reach the beach.

(a) Find the height of the cliff.

(b) With what velocity would the stone have to be thrown vertically downward from the top of the cliff, to land on the beach after 4 seconds?

Working: (a) The stone is dropped so $u = 0$

Only downwards motion so the positive direction is downwards.

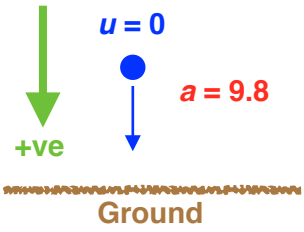
$u = 0, a = 9.8, t = 5, s = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2} \times 9.8 \times 5^2$

$s = 122.5$

The height of the cliff is 122.5 m



(b) $t = 4, a = 9.8, s = 122.5, u = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $122.5 = 4u + \frac{1}{2} \times 9.8 \times 4^2$

$122.5 = 4u + 78.4$

$u = 11.025$

The initial velocity would have to be 11.025 m/s.

E.g. 3 A ball is thrown vertically upward and is caught at the same height 3 seconds later. Find:

(a) the distance it rose

(b) the speed with which it was thrown.

Working: (a) The time to reach the highest point is 1.5 s i.e. half the total time.

Since the ball is moving up and down take the positive direction to be upwards so $a = -9.8$

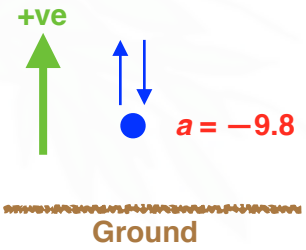
$v = 0, a = -9.8, t = 1.5, s = ?$

No u $\Rightarrow$ $s = vt - \frac{1}{2}at^2$:

$s = 0 - \frac{1}{2} \times (-9.8) \times 1.5^2$

$s = 11.025$

The ball rose 11.025 m.



(b) $v = 0, a = -9.8, t = 1.5, u = ?$

No s $\Rightarrow$ $v = u + at$: $0 = u + (-9.8) \times 1.5$

$u = 14.7$

The speed with which it was thrown was 14.7 m/s.

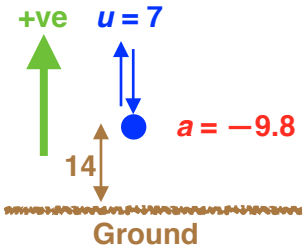
E.g. 4 A ball is thrown vertically with a speed of 7 m/s from a balcony 14 m above the ground.

(a) Find how long it takes to reach the ground if it is thrown

(i) downwards

(ii) upwards.

(b) Find also the speed with which it reaches the ground in each of these cases.



Working: (a) (i) ***Thrown downwards so positive direction is downwards and*** $a = 9.8$

$u = 7, a = 9.8, s = 14, t = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $14 = 7t + \frac{1}{2} \times 9.8 \times t^2$

$4.9t^2 + 7t - 14 = 0$

Solving using a calculator: $t = \frac{-5 \pm \sqrt{165}}{7}$

Since $t \geq 0$, $t \approx 1.12$

It takes 1.12 s (3 s.f.) to reach the ground.

(ii) ***Thrown upwards so positive direction is upwards and*** $a = -9.8$

The displacement is -14 since the positive direction is up.

$u = 7, a = -9.8, s = -14, t = ?$

No v $\Rightarrow$ $s = ut + \frac{1}{2}at^2$: $-14 = 7t + \frac{1}{2} \times (-9.8) \times t^2$

$4.9t^2 - 7t - 14 = 0$

Solving using a calculator: $t = \frac{5 \pm \sqrt{165}}{7}$

Since $t \geq 0$, $t \approx 2.55$

It takes 2.55 s (3 s.f.) to reach the ground.

(b) Thrown downwards: $u = 7, a = 9.8, s = 14, v = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $v^2 = 7^2 + 2 \times 9.8 \times 14$

$v^2 = 323.4$

$v = \frac{7\sqrt{165}}{5} \approx 18.0$

Thrown upwards: $u = 7, a = -9.8, s = -14, v = ?$

No t $\Rightarrow$ $v^2 = u^2 + 2as$: $v^2 = 7^2 + 2 \times (-9.8) \times (-14)$

$v^2 = 323.4$

$v = \frac{7\sqrt{165}}{5} \approx 18.0$

The ball hits the ground with speed 18.0 m/s (3 s.f.) in each case

Video: Vertical motion under gravity

Vertical motion under gravity EQ

Solutions to Starter and E.g.s

Exercise
p469 20C Qu 1i, 2i, 3-8, 10