

Counting problems (Intermediate UKMT)

These questions must be attempted **without a calculator**

Topics covered in the questions below may not necessarily be from the topic of the title.

1. Anna has 3 brothers and 5 sisters. Her brother Tom has S sisters and B brothers.

What is the value of $S \times B$?

A 8 B 10 C 12 D 15 E 18

2. What is the total number of letters in all the incorrect options for this question?

A eleven B twenty two C thirty three D forty four E fifty five

3. In how many whole numbers between 100 and 999 is the middle digit equal to the sum of the other two digits?

A 28 B 36 C 45 D 50 E 55

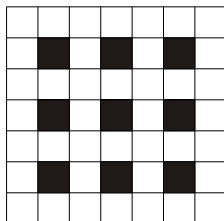
4. You are told that 30 pupils have 25 different birthdays between them.

What is the largest number of these pupils who could share the same birthday?

A 2 B 3 C 4 D 5 E 6

5. The diagram shows a square board in which strips of white squares alternate with strips of black and white squares. A larger board, constructed in the same way, has 49 black squares.

How many white squares are there on the larger board?




A 176 B 196 C 245 D 289 E 392

6. An 8 by 8 chessboard is placed so that a black square is in the top left-hand corner. Starting in the top left square and working along each row in turn, coloured counters are placed, one on each square, following the sequence black, white, red, black, white, red and so on.

When the right-hand end of each row is reached, the pattern continues, starting at the left-hand end of the row beneath, until there is one counter on every square. In the final arrangement, what fraction of the counters are on squares of the same colour as themselves?

- A $\frac{11}{32}$ B $\frac{23}{64}$ C $\frac{7}{16}$ D $\frac{1}{2}$ E $\frac{2}{3}$

7. A square patchwork quilt is made by joining four square pieces of cloth like this  each piece coloured grey and white as shown. Only edges of the same colour are sewn together.

How many different quilt patterns are possible? (Two patterns are considered to be the same if one can be rotated to look exactly like the other.)

- A 3 B 4 C 5 D 6 E 8

8. Wallace and Gromit are waiting in a queue. There are x people behind Wallace, who is y places in front of Gromit.

If there are n people in front of Gromit, how long is the queue?

- A $n - x + y + 2$ B $n + x - y$ C $n - x + y - 1$ D $n + x - y + 1$ E $n - x + y$

9. One hundred and twenty students take an exam which is marked out of 100 (with no fractional marks). No three students are awarded the same mark.

What is the smallest possible number of pairs of students who are awarded the same mark?

- A 9 B 10 C 19 D 20 E 60

10. A digital clock uses two digits to display hours, two digits to display minutes and two digits to display seconds, e.g. 10:23:42.

How many times between 10:00:00 and 11:00:00 on the same morning are all six digits different?

- A 120 B 240 C 360 D 480 E 600