Divisibility (Intermediate UKMT)

These questions must be attempted ***without a calculator***
Topics covered in the questions below may not necessarily be from the topic of the title.

1. The pattern 123451234512345... is continued to form a 2000-digit number.

What is the sum of all 2000 digits?

A 6000 B 7500 C 30 000 D 60 000 E 75 000

2. The standard Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, … begins with two 1s, and each later number in the sequence is the sum of the previous two numbers. Other Fibonacci-like sequences can be constructed by starting with any two numbers *a* and *b* (not necessarily 1 and 1) and using the same rule for creating the other numbers in the sequence.

What is the first term of the Fibonacci-like sequence whose second term is 4 and whose fifth term is 22?

A 2 B 3 C 4 D 5 E 6

3. If the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are all multiplied together, how many zeros are at the end of the answer?

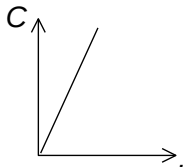
A 1 B 2 C 3 D 4 E 10

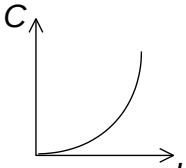
4. Each interior angle of a particular polygon is an obtuse angle which is a whole number of degrees.

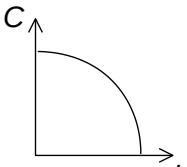
What is the greatest number of sides the polygon could have?

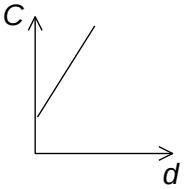
A 90 B 179 C 180 D 359 E 360

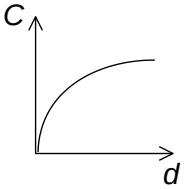
5. Which of the following could be the graph showing the circumference C of a circle in terms of its diameter d?

A 

B 

C 

D 

E 

6. In how many different ways can seven different numbers be chosen from the numbers 1 to 9 inclusive so that the seven numbers have a total which is a multiple of 3?

A fewer than 10 B 10 C 11 D 12 E more than 12

7. The following sequence continues indefinitely:

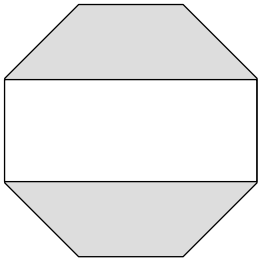
$27 = 3 \times 3 \times 3$ $207 = 3 \times 3 \times 23$ $2\,007 = 3 \times 3 \times 223$ $20\,007 = 3 \times 3 \times 2\,223$, ...

Which of the following integers is a multiple of 81?

A 200 007 B 20 000 007 C 2 000 000 007 D 200 000 000 007 E 20 000 000 000 007

8. Each of the sides of this regular octagon has length 2cm.

What is the difference between the area of the shaded region and the area of the unshaded region (in cm^2)?



A 0 B 1 C 1.5 D 2 E $2\sqrt{2}$