

Combinatorics (Intermediate UKMT)

These questions must be attempted without a calculator

Topics covered in the questions below may not necessarily be from the topic of the title.

1. A *Langford number* is one in which each digit of the number occurs twice; the digits 1 are separated by one other digit, the digits 2 are separated by two others, and so on.

Which of the following is a Langford number?

A 12142334 B 41312432 C 14132342 D 32432141 E 31213244

2. In the game Four-in-a-Row, two players take it in turns to place counters on the 5 × 5 board. The winner is the first player to have four adjacent counters in a line across or down (but not diagonally). It is Black's turn to play next.

	E	○	○	
	●	B		
	C	○		
D	●	●	A	

Where should she play her fourth counter to be certain of winning on her fifth turn whatever White plays?

A B C D E

3. In this unusual game of noughts and crosses the first player to form a line of three Os or three Xs *loses*. It is X's turn.

Where should she place her cross to make sure that she does not lose?

A	O	B
C	X	D
E	X	O

A B C D E

4. Jack had five cards in a pile on a table. He gave me the top card, and then placed the next card at the bottom of the pile; then he gave me the next one on the top and placed the next one after that at the bottom of his pile. He continued like this until he had given me all of the cards. I looked down and to my surprise found that Jack had given me the cards in order: Ace, 2, 3, 4, 5.

In what order (top to bottom) did Jack originally have the cards in the pile on the table?

A Ace, 2,3,4,5 B Ace,4,2,5,3 C Ace,5,2,3,4 D Ace,5,2,4,3 E Ace,5,3,4,2

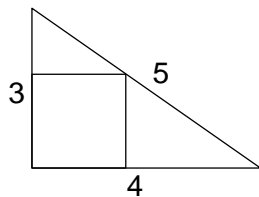
5. If the pattern shown is continued, what number will appear directly below 400?



- A 438 B 439 C 440 D 441 E 442

6. A square is inscribed in a 3-4-5 right-angled triangle as shown.

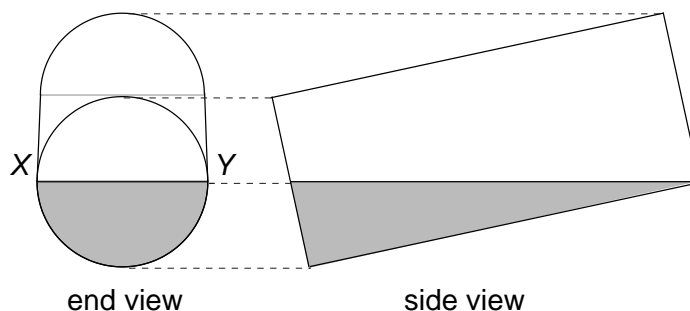
What fraction of the triangle does it occupy?



- A $\frac{12}{25}$ B $\frac{24}{49}$ C $\frac{1}{2}$ D $\frac{25}{49}$ E $\frac{13}{25}$

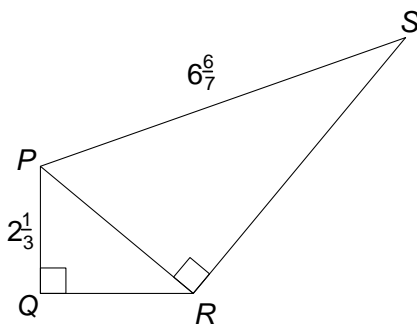
7. A cylindrical can contains lemonade, shown shaded on the diagram in which XY is a diameter.

What fraction of the volume of the can is filled with lemonade?



- A just below a quarter B just above a quarter C exactly a quarter
 D just below a half E exactly a half

8. In the figure below, $PQ = 2\frac{1}{3}$, $PS = 6\frac{6}{7}$ and $\angle QPR = \angle RPS$. How long is PR ?



- A $3\frac{1}{2}$ B 4 C $4\frac{1}{4}$ D $4\frac{25}{42}$ E 5