Counting problems 2 (Junior UKMT)

These questions must be attempted *without a calculator*

Topics covered in the questions below may not necessarily be from the topic of the title.

1. The equilateral triangle *XYZ* is fixed in position. Two of the four small triangles are to be painted black and the other two are to be painted white.

In how many different ways can this be done?



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2. A station clock shows each digit by illuminating up to seven bars in a display. For example, the displays for 1, 6, 4 and 9 are shown.

When all the digits from 0 to 9 are shown in turn, which bar is used least?



3. The Kings of Clubs, Diamonds, Hearts and Spades, and their respective Queens, are having an arm wrestling competition. Everyone must wrestle everyone else, except that no King will wrestle his own Queen.

How many wrestling bouts are there?

6.

	A 12	B 16	C 24	D 28	E 64
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4. Among the children in a certain family, each child has at least one brother and at least one sister.

What is the smallest possible number of children in the family?

A 2 B 3 C 4 D 5 E 6

5. How many triangles of any size are there in this diagram?

How many of the five words above can be placed in the gap to make the sentence in the box true?

A 0 B 1 C 2 D 3 E 4

7. The faces of a cube are painted so that any two faces which have an edge in common are painted different colours.

What is the smallest number of colours required?

- A 2 B 3 C 4 D 5 E 6
- **8.** Jack dances clockwise around the Maypole, making one revolution every five seconds. Starting from a point diametrically opposite Jack's starting point, Jill dances anticlockwise, making one revolution every six seconds.

How many times do they pass each other in the first minute?





9. In how many different ways can a row of five "on/off" switches be set so that no two adjacent switches are in the "off" position?

A 5 B 10 C 11 D 13 E 15

- **10.** If all the whole numbers from 1 to 1000 inclusive are written down, which digit appears the smallest number of times?
 - A 0 B 2 C 5 D 9 E none: no single digit appears fewer times than all the others
- **11.** Sixty 20p coins are lined up side by side. Every second 20p coin is then replaced by a 10p coin. Then every third coin in the resulting row is replaced by a 5p coin. Finally every fourth coin in the row is replaced by a 2p coin.

What is the final value of the line?

A £3.30 B £5.80 C £6.05 D £6.60 E £7.55

12. The figure shows rectangle *PRSU* and line *QT*, which divides the rectangle into two squares.

How many right-angled triangles can be drawn using any three of the points P, Q, R, S, T, U as corners?

