Sequences (Senior UKMT)

These questions must be attempted ***without a calculator***

Topics covered in the questions below may not necessarily be from the topic of the title.

1. The number 2005 is the sum of a sequence of five consecutive positive integers.

Which of the following numbers occurs in this sequence?

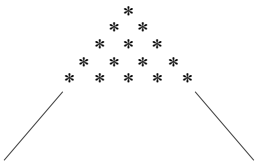
A 395 B 400 C 405 D 410 E 415

2. The mean of seven consecutive odd numbers is 21. What is the sum of the first, third, fifth and seventh of these numbers?

A 16 B 21 C 84 D 147 E more information needed

3. A gardener has a field in the shape of an isosceles triangle. He decided to plant it with rose bushes with the aid of his apprentice. He planted the first row with 101 bushes along the base, then his apprentice planted the next row with 100 bushes. They continued in this way, planting alternate rows, until the whole field was planted. The gardener planted the last row, which contained 1 bush.

How many more bushes did the gardener plant than the apprentice?



A 100 B 101 C 49 D 50 E 51

4. Consider the arithmetic sequences 1998, 2005, 2012,…and 1996, 2005, 2014,….
Which is the next number after 2005 that appears in both sequences?

A 2054 B 2059 C 2061 D 2063 E 2068

5. Two square pieces of card, each 3 cm × 3 cm, are attached by a single pin to a board. The pin passes through a point 1/3 of the way along the diagonal of each square and the squares overlap exactly. The bottom card now remains fixed, while the top card is rotated through 180°.

What is the area of overlap of the cards in this new position?

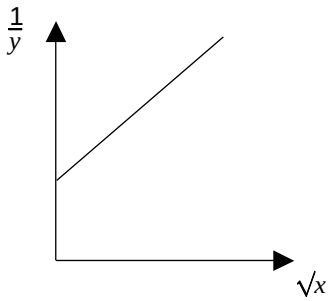
A 1 cm^2 B 2 cm^2 C 4 cm^2 D 6 cm^2 E 9 cm^2

6. The line whose equation is $y = 3x + 4$ is reflected in the line whose equation is $y = -x$.

What is the equation of the image line?

A $3y = x + 4$ B $3y = x - 4$ C $y = 3x - 4$ D $y = -3x - 4$ E $y = 4x + 3$

7. The straight line shows the graph of $\frac{1}{y}$ plotted against $\sqrt{x}$.



Which of the following could be a possibility for the equation linking y and x?

A $y^2 = \frac{1}{x-1}$ B $y^2 = \frac{1}{x^2+1}$ C $y^2 = x-1$

D $y^2 = \frac{1}{x-2\sqrt{x}+1}$ E $y^2 = \frac{1}{x+2\sqrt{x}+1}$

8. The value of $1^{2004} + 3^{2004} + 5^{2004} + 7^{2004} + 9^{2004}$ is calculated using a powerful computer.

What is the units digit of the correct answer?

A 9 B 7 C 5 D 3 E 1

9. Which of the following straight lines should be omitted to leave four lines which determine a square?

A $y + x = 3$ B $y = x - 1$ C $y + x = 1$ D $y = x + 1$ E $y + x = 2$

10. What is the value of $2^{2003} - 2^{2002} - 2^{2001} - 2^{2000}$?

A -2^{2002} B 0 C 2^{-4000} D 64 E 2^{2000}

11. Given that $S = (x + 20) + (x + 21) + (x + 22) + \ldots + (x + 100)$, where x is a positive integer, what is the smallest value of x such that S is a perfect square?

A 1 B 2 C 4 D 8 E 64

12. Which positive integer n satisfies the equation

$$\frac{3}{n^3} + \frac{4}{n^3} + \frac{5}{n^3} + \ldots + \frac{n^3-5}{n^3} + \frac{n^3-4}{n^3} + \frac{n^3-3}{n^3} = 60?$$

A 5 B 11 C 31 D 60 E 2006