

Fractions and shape (Senior UKMT)

These questions must be attempted **without a calculator**

Topics covered in the questions below may not necessarily be from the topic of the title.

1. What is the value of $\frac{2007}{9} + \frac{7002}{9}$?
- A 500.5 B 545 C 1001 D 1655 E 2007
2. Travelling at an average speed of 100 km/hr, a train took 3 hours to travel to Birmingham. Unfortunately the train then waited just outside the station, which reduced the average speed for the whole journey to 90 km/hr.
- For how many minutes was the train waiting?
- A 1 B 5 C 10 D 15 E 20
3. A ball is dropped out of a classroom window onto the playground 29 feet 3 inches below.
- Every time the ball hits the ground it bounces to two thirds of its previous height.
- What is the greatest height to which it rises following the third bounce?
- A 1 foot 1 inch B 3 feet 3 inches C 8 feet 8 inches
D 13 feet E 13 feet 5 inches
4. Sam can mow a lawn in 3 hours. Mel takes 4 hours to mow the same lawn, and Chris takes 6 hours to do the same.
- If they work with a lawn mower each, and do not get in the way of each other, how long would they take to mow the lawn together?
- A 1 hour 20 minutes B 1 hour 30 minutes C 3 hours
D 4 hours 20 minutes E 13 hours
5. What is the value of the expression: $(1 + \frac{1}{2})(1 + \frac{1}{3})(1 + \frac{1}{4}) \dots (1 + \frac{1}{2004})(1 + \frac{1}{2005})$?
- A 1001 B 1002 C 1003 D 1004 E 1005
6. The base of a pyramid has n edges.
- What is the difference between the number of edges the pyramid has and the number of faces the pyramid has?
- A $n - 2$ B $n - 1$ C n D $n + 1$ E $n + 2$

7. An engineer is directed to a faulty signal, one quarter of the way into a tunnel. Whilst there, he is warned of a train heading towards the tunnel entrance. The engineer can run at 12 mph and can either run back to the tunnel entrance or forward to the exit. In either case, the engineer and the front of the train would reach the entrance or exit together.

What is the speed in mph of the train?

- A 16 B 20 C 24 D 32 E more information needed

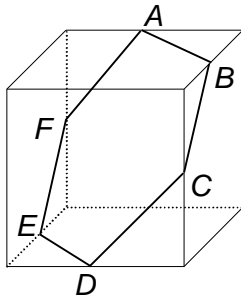
8. A cube exactly fits inside a sphere and another sphere exactly fits inside this cube.

What is the ratio of the volume of the smaller sphere to the volume of the larger sphere?

- A $1 : 3\sqrt{3}$ B $1 : 4$ C $1 : 3$ D $2 : 3$ E $1 : 2$

9. A solid red plastic cube, volume 1cm^3 , is painted white on its outside. The cube is cut by a plane passing through the midpoints of various edges, as shown.

What, in cm^2 , is the *total* red area exposed by the cut?



- A $\frac{3\sqrt{3}}{2}$ B 2 C $\frac{9\sqrt{2}}{5}$ D 3 E $\frac{3(\sqrt{3} + \sqrt{2})}{4}$

10. A paperweight is made from a glass cube of side 2 units by first shearing off the eight tetrahedral corners which touch at the midpoints of the edges of the cube. The remaining inner core of the cube is discarded and replaced by a sphere. The eight corner pieces are now stuck onto the sphere so that they have the same positions relative to each other as they did originally.

What is the diameter of the sphere?



- A $\sqrt{8} - 1$ B $\sqrt{8} + 1$ C $\frac{1}{3}(6 + \sqrt{3})$ D $\frac{4}{3}\sqrt{3}$ E $2\sqrt{3}$