

Miscellaneous (Senior UKMT)

These questions must be attempted without a calculator

Topics covered in the questions below may not necessarily be from the topic of the title.

1. I am trying to do a rectangular jigsaw puzzle. The puzzle was made by starting with a rectangular picture and then cutting it into 1000 pieces by sawing along the lines of a (wiggly!) rectangular grid. I start by separating out all the edge and corner pieces.

Which of the following could *not* possibly be the number of corner and edge pieces of such a jigsaw?

A 126 B 136 C 216 D 316 E A-D are all possible

2. What is the remainder when $123456789 \times 987654321$ is divided by 6?

A 1 B 2 C 3 D 4 E 5

3. Cheryl finds a bag of coins. There are 50 coins inside and the value of the contents is £1.81.

Given that it contains only two-pence and five-pence coins, how many more five-pence coins are there inside the bag than two-pence coins?

A 4 B 6 C 8 D 10 E 12

4. A square number is divided by 6.

Which of the following could not be the remainder?

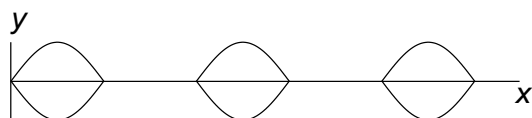
A 0 B 1 C 2 D 3 E 4

5. What is the greatest number of the following five statements about numbers a, b which can be true at the same time?

$\frac{1}{a} < \frac{1}{b}$ $a^2 > b^2$ $a < b$ $a < 0$ $b < 0$

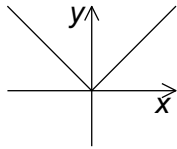
A 1 B 2 C 3 D 4 E 5

6. Which of the following equations could be the equation of the "curve", part of which is shown here?

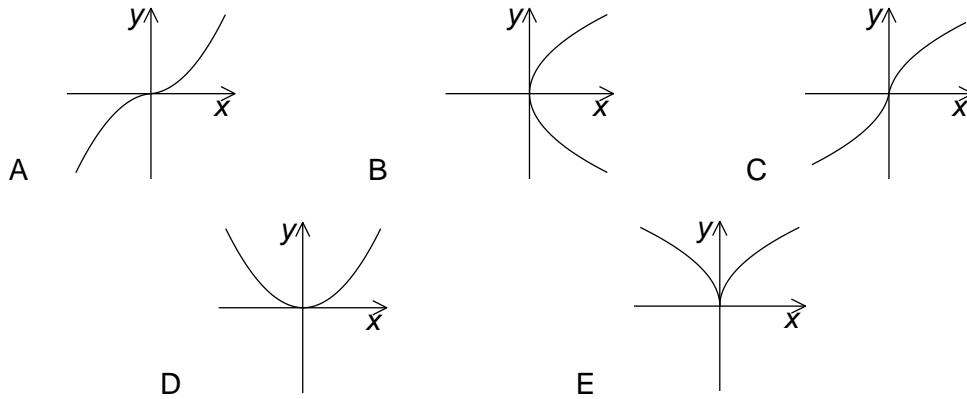


A $y = \sin x$ B $|y| = \sin x$ C $y = |\sin x|$ D $|y| = |\sin x|$ E A-D all impossible

7. The graph of $y = |x|$ is shown below.

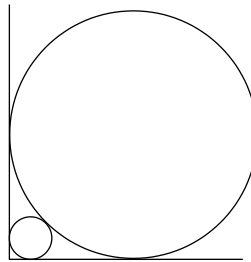


Which of the following could be a sketch of the graph of $y = x|x|$?



8. Circles with radii r and R (where $r < R$) touch each other and also touch two perpendicular lines as shown.

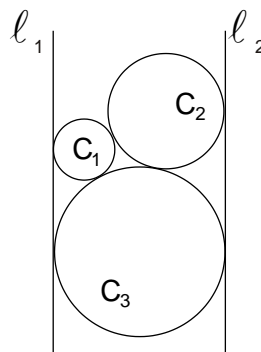
What is the value of R/r ?



- A $5 + \sqrt{2}$ B 5.75 C $\sqrt{40}$ D 6 E $3 + 2\sqrt{2}$

9. The figure shows two parallel lines, l_1 , and l_2 . Line l_1 is a tangent to circles C_1 and C_3 , line l_2 is a tangent to circles C_2 and C_3 and the three circles touch as shown. Circles C_1 and C_2 have radius s and t respectively.

What is the radius of circle C_3 ?



- A $2\sqrt{s^2 + t^2}$ B $s + t$ C $2\sqrt{st}$ D $\frac{4st}{s+t}$
 E more information needed

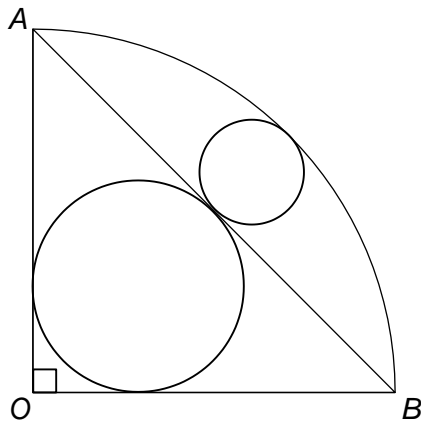
10. A square $XABD$ of side length 1 is drawn inside a circle with diameter XY of length 2. The point A lies on the circumference of the circle. Another square $YCBE$ is drawn.

What is the ratio of the area of square $XABD$ to the area of square $YCBE$?

- A 1 : 2 B 1 : 3 C $1 : \sqrt{2}(2 - \sqrt{3})$ D $1 : (\sqrt{2} - 1)$ E $1 : (2 - \sqrt{3})$

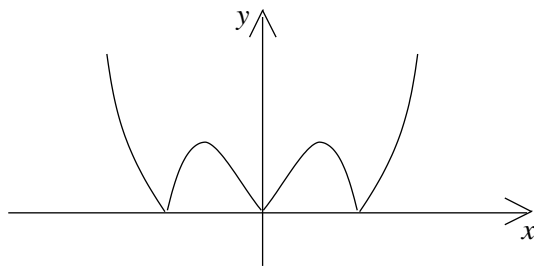
11. AOB is an isosceles right-angled triangle drawn in a quadrant of a circle of radius 1 unit. The largest possible circle is drawn in the minor segment cut off by the line AB . This circle has radius r . The radius of the inscribed circle of the triangle AOB is R .

What is the value of $\frac{R}{r}$?



- A 2 B $2\sqrt{2} - 1$ C $\sqrt{2} + 1$ D $\frac{9}{5}$ E $\sqrt{3}$

12. The graph of $y = |f(x)|$ is shown. Given that the graph of $y = f(x)$ is a continuous curve, how many different possibilities are there for the graph of $y = f(x)$?



- A 16 B 12 C 8 D 4 E 2