

Trigonometry (Senior UKMT)

These questions must be attempted without a calculator

Topics covered in the questions below may not necessarily be from the topic of the title.

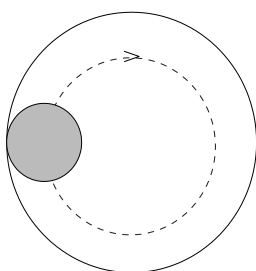
1. The sum of five consecutive even numbers is 60. What is the smallest of the five numbers?
- A 8 B 10 C 12 D 14 E 16
2. The *integer part* of a positive number is the part before the decimal point; the *decimal part* is the part after the decimal point. For example, the integer part of 3.72 is '3' and the decimal part is '0.72'.

Which of the following numbers has decimal part equal to exactly one eighth of the integer part?

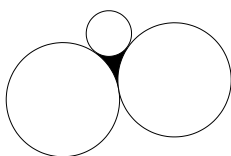
- A 0.05 B 1.15 C 2.25 D 3.35 E 4.45

3. A circular disc of diameter d rolls without slipping around the inside of a ring of internal diameter $3d$, as shown in the diagram.

By the time that the centre of the inner disc returns to its original position for the first time, how many times will the inner disc have turned about its centre?



- A 1 B π C 3 D 2π E 2
4. If $\cos \theta = 1/2$, which of these cannot equal $\sin 2\theta$?
- A $\sin \theta$ B $1/2$ C $-\sqrt{3}/2$ D $\sqrt{3}/2$ E $2 \cos \theta \sin \theta$
5. Three circles touch, as shown in the diagram. The two larger circles both have radius 1 and the smaller circle has radius $\sqrt{2} - 1$.



What is the perimeter of the shaded region?

- A $\frac{\pi}{4}(\sqrt{2} - 1)$ B $\frac{\pi}{2}(\sqrt{2} - 1)$ C $\frac{\pi}{2}$ D $\frac{\pi}{4}(\sqrt{2} + 1)$ E $\frac{\pi}{\sqrt{2}}$

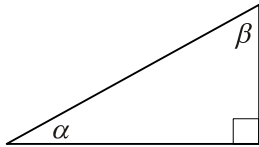
6. All six vertices of hexagon $UVWXYZ$ lie on the circumference of a circle; $\angle ZUV = 88^\circ$ and $\angle XYZ = 158^\circ$.

What is the size of $\angle VWX$?

- A 92° B 114° C 120° D 132° E it is impossible to determine

7. If $\alpha < \beta$, how many different values are there among the following expressions?

$$\sin \alpha \sin \beta \quad \sin \alpha \cos \beta \quad \cos \alpha \sin \beta \quad \cos \alpha \cos \beta$$



- A 1 B 2 C 3 D 4 E It depends on the value of α

8. Which of the following expressions is identically equal to $\sin^3 x + \cos^3 x$?

A $\sin 3x + \cos 3x$ B 1 C $(\sin x + \cos x)(1 - \sin x \cos x)$

D $(\sin x + \cos x)^3$ E $(\sin x + \cos x)(2 \sin x \cos x + 1)$

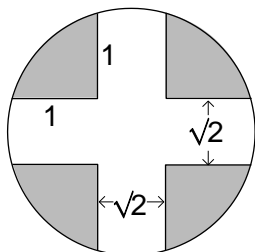
9. A right circular cone has apex angle 2α . A sphere is inscribed in the cone, touching the base. What fraction of the cone is occupied by the sphere?

A $2 \sin \alpha \cos 2\alpha / (1 + \cos \alpha)^3$ B $4 \sin \alpha (1 - \sin \alpha) / \cos^2 \alpha$ C $4(1 - \sin \alpha)^3 \sin \alpha / \cos^4 \alpha$

D $\sin 2\alpha \cos \alpha / (1 + \sin \alpha)^3$ E need more information

10. A company logo has a centrally-symmetric white cross of width $\sqrt{2}$ on a dark circle. The dark corner pieces have sides of length 1 as indicated.

What is the total area of the corners?



A $\pi(2 - \sqrt{2}) + \frac{\sqrt{2}}{2}$

B $\pi - \frac{1}{\sqrt{2}}$

C $\pi(4 - \sqrt{2}) - 4\sqrt{2}$

D $\frac{(\pi + \sqrt{2})}{2}$

E $\frac{\pi(2 + \sqrt{2})}{2} - 2\sqrt{2}$