Factors (Senior UKMT)

These questions must be attempted ***without a calculator***

Topics covered in the questions below may not necessarily be from the topic of the title.

1. Goldbach's conjecture, which has not been proved, states that every even number greater than two is the sum of two primes. However, the same is not true for every odd number.

Which of the following odd numbers is *not* the sum of two primes?

A 13 B 33 C 43 D 53 E 73

2. Pat's age is a prime number. Ten years ago, as a teenager, Pat's age was also a prime number.

How old is Pat?

A 17 B 23 C 27 D 29 E more information needed

3. Which symbol should replace $\oplus$ to make the following equation true?

$1 \times 2 \times (3 \oplus 4 + 5) \times (6 \times 7 + 8 + 9) = 2006.$

A + B − C ÷ D × E none of these

4. The number of the year 2003 is prime.

How many square numbers are factors of 2003^{2003}?

A 0 B 1 C 44 D 1002 E 2003

5. Damien wishes to find out if 457 is a prime number. In order to do this he needs to check whether it is exactly divisible by some prime numbers. What is the smallest number of possible prime number divisors that Damien needs to check before he can be sure that 457 is a prime number?

A 8 B 9 C 10 D 11 E 12

6. Three people each think of a number which is the product of two different primes.

Which of the following could be the product of the three numbers which are thought of?

A 120 B 144 C 240 D 3000 E 12100

7. What is the 1999th term of the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, … ?

A 59 B 60 C 61 D 62 E 63

8. Just one of the following is a prime number.

Which one is it?

A $1000^2 + 111^2$ B $555^2 + 666^2$ C $2000^2 - 999^2$

D $1001^2 + 1002^2$ E $1001^2 + 1003^2$

9. The statement "There are exactly four integer values of n for which $(2n + y)/(n - 2)$ is itself an integer" is true for certain values of y only.

For how many values of y in the range $1 \le y \le 20$ is the statement true?

A 0 B 7 C 8 D 10 E 20

10. What is the sum to infinity of the convergent series
$\frac{1}{2}+\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{5}{32}+\frac{8}{64}+\frac{13}{128}+\frac{21}{256}+\frac{34}{512}+...?$

A $\frac{7}{4}$ B 2 C $\sqrt{5}$ D $\frac{9}{4}$ E $\frac{7}{3}$