

## Algebra (Senior UKMT)

These questions must be attempted without a calculator

Topics covered in the questions below may not necessarily be from the topic of the title.

1. If  $a \oplus b = \sqrt{ab+4}$  then what is the value of  $(2 \oplus 6) \oplus 8$ ?  
A 6                      B 8                      C 10                      D 12                      E 18
  
2. What is the value of  $2006 \times 2008 - 2007 \times 2007$ ?  
A -2007              B -1                      C 0                      D 1                      E 4 026 042
  
3. A jogger runs a certain distance at  $V \text{ ms}^{-1}$ , and then walks half that distance at  $U \text{ ms}^{-1}$ .  
If the total time for the two stages is  $T$  seconds, what is the total distance travelled (in metres)?  
A  $\frac{3TUV}{U+2V}$       B  $\frac{3TUV}{2U+V}$       C  $\frac{3T}{U+2V}$       D  $\frac{TUV}{2U+V}$       E  $\frac{2TUV}{2U+V}$
  
4. Given that  $a$  and  $b$  are integers greater than zero, which of the following equations could be true?  
A  $a - b = a \div b$       B  $a + b = a \div b$       C  $a - b = a \times b$   
D  $a + b = a - b$       E  $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$
  
5. For each real number  $x$ , except  $x = 0, 1, -1$ , let  $f(x) = \frac{(x-1)}{(x+1)}$ .  
Then  $f^6(x) = f(f(f(f(f(f(x)))))$  equals  
A  $\frac{(x+1)}{(x-1)}$       B  $\frac{-1}{x}$                       C  $x$                       D  $\frac{1}{x}$                       E  $\left[ \frac{x-1}{x+1} \right]$
  
6.  $(x-1)(x^4+1)(x^2+1)(x+1)$  equals  
A  $x^8 - 1$                       B  $x^8 + x^6 + x^4 + x^2 + 1$                       C  $x^8 + 1$   
D  $x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$                       E  $x^8 - x^6 + x^4 - 1$

7. The following equation is true for all  $a$ ,  $b$  and  $c$ :  
 $a^3 + b^3 + c^3 = (a + b + c)^3 - 3(a + b + c)(ab + bc + ca) + kabc$
- What is the value of  $k$ ?
- A -6            B -3            C 0            D 3            E 6
8. Given that  $x = \frac{1}{y}$ , where  $x$  and  $y$  are unequal and non-zero, which of the following is always equal to  $\left(x + \frac{1}{x}\right)\left(y - \frac{1}{y}\right)$ ?
- A  $y^2 - x^2$     B  $x^2 - y^2$     C  $2y$             D  $2x$             E 0
9. Which of the following is divisible by 3 for every whole number  $x$ ?
- A  $x^3 - x$         B  $x^3 - 1$         C  $x^3$             D  $x^3 + 1$         E  $x^3 + x$
10. Given that  $0 < b < a$  and  $a^2 + b^2 = 6ab$ , what is the value of  $\frac{a+b}{a-b}$ ?
- A  $\frac{1}{\sqrt{2}}$             B  $\sqrt{2}$             C  $\frac{1}{\sqrt{2}-1}$         D  $2\sqrt{2}$             E  $\sqrt{6}$
11. How many pairs of positive integers  $(x, y)$  are solutions of the equation  $\frac{1}{x} + \frac{2}{y} = \frac{3}{19}$ ?
- A 0            B 1            C 2            D 3            E more than 3
12.  $X$  is a positive integer in which each digit is 1; that is,  $X$  is of the form 11111... .  
 Given that every digit of the integer  $pX^2 + qX + r$  (where  $p$ ,  $q$  and  $r$  are fixed integer coefficients and  $p > 0$ ) is also 1, irrespective of the number of digits in  $X$ , which of the following is a possible value of  $q$ ?
- A -2            B -1            C 0            D 1            E 2