Shape (Senior UKMT)

These questions must be attempted ***without a calculator***

Topics covered in the questions below may not necessarily be from the topic of the title.

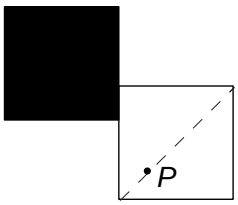
1. A furlong is 220 yards long and a yard is 36 inches. A chain is 44 cubits long and a cubit is 54 barleycorns. There are 10 chains in a furlong.

How many barleycorns are there in one inch?

A 6 B 5 C 4 D 3 E 2

2. A square piece of wood, of side 8 cm, is painted black and fixed to a table. An equal square, painted white, is placed on the table alongside the black square and has a point *P* marked one quarter of the way along a diagonal, as shown. Whilst keeping the same orientation on the table and always remaining in contact with the black square, the white square now slides once around the black square.

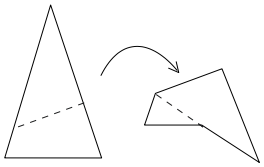
Through what distance does *P* move?



A 32 cm B 48 cm C 64 cm D 72 cm E 80 cm

3. The diagram shows a triangular piece of paper that has been folded once to produce a shape with the outline of a pentagon.

If a *rectangular* piece of paper is folded once, what is the smallest value of *n* (greater than 4) for which it is not possible to create an *n*-sided polygon in the same way?



A 6 B 7 C 8 D 9 E 10

4. One face of a solid polyhedron is a regular hexagon.

What is the smallest possible number of edges the polyhedron could have?

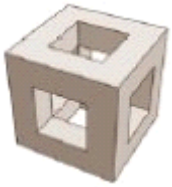
A 7 B 9 C 12 D 15 E 18

5. How many differently shaped triangles exist in which no two sides are the same length, each side is of integral unit length and the perimeter of the triangle is less than 13 units?

A 2 B 3 C 4 D 5 E 6

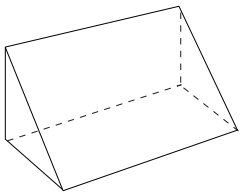
6. A 4 × 4 × 4 cube has three 2 × 2 × 4 holes drilled symmetrically all the way through, as shown.

What is the surface area of the resulting solid?



A 192 B 144 C 136 D 120 E 96

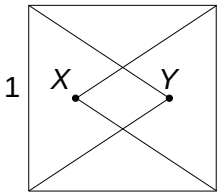
7. Which shape cannot be obtained as the cross-section (in any direction) of this solid, which is a triangular prism with three rectangular faces?



A triangle B rectangle C trapezium D pentagon E hexagon

8. The diagram shows a square and two equilateral triangles. All the sides have length 1.

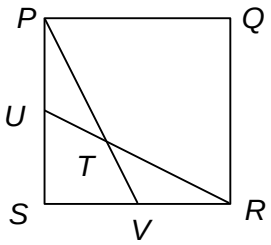
What is the length of *XY*?



A $\sqrt{3}-1$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{\sqrt{3}}{2}$ E $\frac{2+\sqrt{3}}{4}$

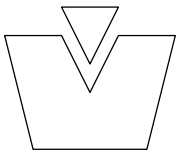
9. *PQRS* is a square with *U* and *V* the mid-points of the sides *PS* and *SR* respectively. Line segments *PV* and *UR* meet at *T*.

What fraction of the area of the square *PQRS* is the area of the quadrilateral *PQRT*?



A $\frac{1}{2}$ B $\frac{5}{8}$ C $\frac{2}{3}$ D $\frac{3}{4}$ E $\frac{5}{9}$

10. The expression "$3 \oplus 7 \rightarrow 4$ " is a short way of writing the statement "it is possible to fit a 3-sided polygon and a 7-sided polygon together (without overlap) and so make a 4-sided polygon". This statement is correct (as shown in the diagram below).

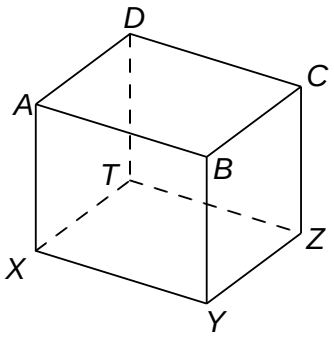


Which of the following represents a statement which is *not* correct?

A $3 \oplus 5 \rightarrow 4$ B $3 \oplus 6 \rightarrow 4$ C $3 \oplus 8 \rightarrow 4$ D $4 \oplus 6 \rightarrow 4$ E $4 \oplus 8 \rightarrow 4$

11. The cube, *XYZTABCD*, is cut into four pieces by cutting along two planes, *BCTX* and *BDTY*.

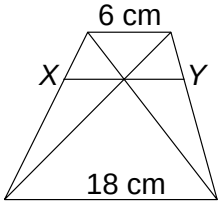
What fraction of the volume of the cube is occupied by the piece containing corner *A*?



A $\frac{3}{8}$ B $\frac{1}{3}$ C $\frac{3}{10}$ D $\frac{5}{18}$ E $\frac{1}{4}$

12. In the trapezium shown (not to scale), *XY* is parallel to two sides and passes through the point of intersection of the diagonals.

What is the length *XY*?



A 8 cm B 9 cm C 10 cm D 11 cm E 12 cm